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## EMAP5: A 3D HYBRID FEM/MOM CODE

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**Abstract**--EMAP5 is a numerical software package designed to model electromagnetic problems. It employs the finite element method (FEM) to analyze a volume, and employs the method of moments (MoM) to analyze the region exterior to the FEM volume. The two methods are coupled by enforcing the continuity of the tangential fields on the interface. Dielectrics, lossy dielectrics and metal objects can be modeled within the FEM volume. Metal objects can also be located exterior to, or on the surface of the FEM volume. A simple input file translator (SIFT) is provided with EMAP5 that allows users to define basic input configurations on a rectangular grid without a graphical interface. It is also possible to use commercial mesh generators to analyze more complex geometries of arbitrary shape. EMAP5 can model three types of sources: incident plane waves, voltage sources on metal patches and impressed current sources in the finite element region. This paper describes the implementation of EMAP5 and provides four examples to demonstrate the range of configurations that EMAP5 is capable of modeling using the SIFT translator.

### I. INTRODUCTION

Methods used to solve Maxwell's equations can often be classified into one of two categories. One category is integral equation-based methods. Integral equation-based methods generally solve for unknown fields or currents on the surface of homogeneous volumes. Only the surfaces are meshed; therefore the number of unknowns is relatively small. Integral equation-based methods are well suited for modeling homogeneous structures in unbounded geometries.

The other category is partial differential equation (PDE)-based methods. PDE-based methods require a volume mesh. The resulting number of unknowns is usually much higher than that obtained from integral equation-based methods, but the reduced complexity of the solution method can more than compensate for this. PDE-based methods are well suited for modeling complex inhomogeneous structures, particularly those in bounded geometries (e.g. a waveguide or a metallic enclosure).

The finite element method (FEM) is a powerful PDE-based method. It requires the entire region of interest to be meshed. However, each mesh element can have its own electromagnetic properties ( $\epsilon$ ,  $\mu$ ,  $\sigma$ ).

Thus, FEM has an inherent capacity to analyze inhomogeneous structures. In addition, FEM does not require a uniform mesh. Various types of elements, for example, bricks, tetrahedra and prisms, can be used to discretize arbitrary geometric configurations. Therefore, FEM is well suited for modeling complex, inhomogeneous structures. A significant disadvantage of the FEM method is the inability to model unbounded geometries without using absorbing boundary elements. Absorbing boundary elements are not effective when they closely conform to the shape of long thin objects (e.g. cables). When modeling objects with one dimension significantly longer than the others using FEM, it is generally necessary to fill the empty space around these objects with many elements that transition from the object of interest to the absorbing boundary. This can significantly inflate the overall size of the problem.

The method of moments is not actually an electromagnetic modeling technique, but rather it is a technique for solving complex linear equations [1, 2]. However, electromagnetic modeling techniques that apply the method of moments to the solution of electric or magnetic field integral equations are generally referred to as method-of-moment (MoM) techniques. MoM techniques can be used to model metallic (particularly wire) structures in unbounded geometries very effectively.

In order to take advantage of the strengths of both the FEM and MoM methods, several researchers have developed hybrid FEM/MoM approaches [3-6]. Hybrid FEM/MoM methods generally take advantage of the FEM's ability to model complex inhomogeneous structures and the MoM's ability to model unbounded geometries. The specific strengths of a particular code depend on the details of the implementation, particularly the form of the integral equation and the basis and weighting functions employed.

EMAP5 (ElectroMagnetic Analysis Program version 5) is a hybrid FEM/MoM modeling code developed at the University of Missouri-Rolla (UMR). EMAP5 was developed in order to demonstrate and validate the hybrid technique described in [6,7]. This technique, used in conjunction with a mesh generator, is particularly useful for analyzing EMI from printed circuit board geometries with enclosures or attached cables. A rudimentary mesh generator (SIFT5) is distributed with the EMAP5 code. This non-graphical

user interface is simple and portable, making it a good tool for educators and people who want to learn to use the EMAP5 code. However, the meshes generated by the SIFT5 code do not exploit the primary advantage of FEM modeling codes (i.e. the ability to adapt the size and shape of the mesh to fit the geometry). The EMAP5 and SIFT5 source codes can be obtained via the web at <<http://www.emclab.umn.edu/emap5>>.

## II. FORMULATION

Although details of the EMAP5 formulation are described in [6,7], a brief summary is provided below. The general structure of interest is shown in Figure 1. A dielectric volume  $V_2$  has electrical properties ( $\epsilon_2, \mu_2$ ) and is enclosed by a surface  $S_2$ . A conductive volume  $V_3$  is enclosed by a conductive surface  $S_c$ . The fields within  $V_3$  vanish.  $V_1$  denotes the volume outside of  $V_2$  and  $V_3$ , which is assumed to be free space. ( $\mathbf{E}_1, \mathbf{H}_1$ ) and ( $\mathbf{E}_2, \mathbf{H}_2$ ) denote the electric and magnetic fields in  $V_1$  and  $V_2$ , respectively. The unit normal vectors for  $S_2$  and  $S_c$  are defined pointing outward toward  $V_1$ . The structure is excited by an incident wave ( $\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}$ ) or impressed sources ( $\mathbf{J}^{\text{int}}, \mathbf{M}^{\text{int}}$ ). The scattered electric and magnetic fields are ( $\mathbf{E}^s, \mathbf{H}^s$ ). The objective is to solve for the scattered fields ( $\mathbf{E}^s, \mathbf{H}^s$ ) or the surface electric current density on  $S_c$ .

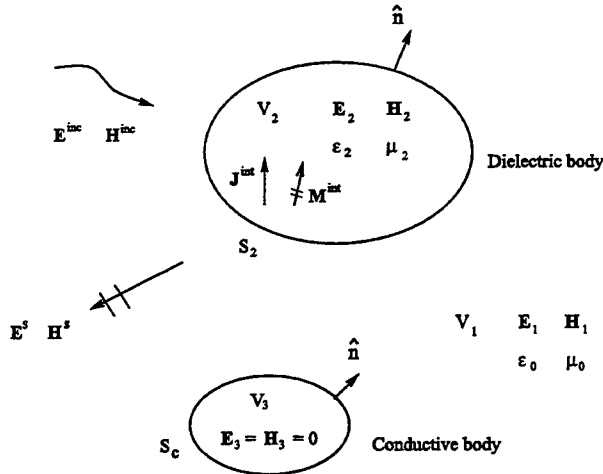


Figure 1. A dielectric object and a conductive object illuminated by ( $\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}$ ) or ( $\mathbf{J}^{\text{int}}, \mathbf{M}^{\text{int}}$ ).

From Maxwell equations, the vector Helmholtz equation in terms of  $\mathbf{E}$  can be written as,

$$\begin{aligned} \nabla \times \left( \frac{\nabla \times \mathbf{E}(\mathbf{r})}{j\omega\mu_0\mu_r} \right) + j\omega\epsilon_0\epsilon_r\mathbf{E}(\mathbf{r}) \\ = -\mathbf{J}^{\text{int}}(\mathbf{r}) - \frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{M}^{\text{int}}(\mathbf{r}). \end{aligned} \quad (1)$$

After multiplying Equation (1) by a weighting function  $\mathbf{w}(\mathbf{r})$  and integrating over the finite element domain  $V_2$ , one obtains the FEM weak form as follows [8, 9],

$$\begin{aligned} \int_{V_2} \left[ \frac{(\nabla \times \mathbf{E}(\mathbf{r})) \cdot (\nabla \times \mathbf{w}(\mathbf{r}))}{j\omega\mu_0\mu_r} + j\omega\epsilon_0\epsilon_r\mathbf{E}(\mathbf{r}) \cdot \mathbf{w}(\mathbf{r}) \right] dV \\ = \int_{S_2} (\hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r})) \cdot \mathbf{w}(\mathbf{r}) dS \\ - \int_{V_2} \left[ \mathbf{J}^{\text{int}}(\mathbf{r}) + \frac{1}{j\omega\mu_0\mu_r} \nabla \times \mathbf{M}^{\text{int}}(\mathbf{r}) \right] \cdot \mathbf{w}(\mathbf{r}) dV \end{aligned} \quad (2)$$

Equation (2) can also be derived using the variational method [10] (pp. 158-161), [11] (pp. 158-160).

Within volume  $V_2$ , EMAP5 uses the vector tetrahedral elements proposed by Barton and Cendes [10] (pp. 56-59), [12]. Each basis function is defined within a tetrahedron, and is associated with one of the six edges. The electric field  $\mathbf{E}$  within volume  $V_2$  can be expanded as,

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{N_v} E_n \mathbf{w}_n(\mathbf{r}) \quad (3)$$

where  $N_v$  is the total number of interior edges and boundary edges on the dielectric surface, and  $\{E_n\}$  is a set of unknown complex scalar coefficients.

The surface integral term in Equation (2) can be evaluated by using a surface basis function  $\mathbf{f}(\mathbf{r})$ , which is discussed later. The weighting functions chosen are  $\mathbf{w}_n(\mathbf{r})$ ,  $n=1 \dots N_v$ . After multiplying by the weighting functions, Equation (2) can be discretized as,

$$\begin{bmatrix} A_{ii} & A_{id} \\ A_{di} & A_{dd} \end{bmatrix} \begin{bmatrix} E_i \\ E_d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B_{dh} \end{bmatrix} \begin{bmatrix} 0 \\ J_h \end{bmatrix} + [g^{\text{int}}] \quad (4)$$

The unknown coefficients  $[E_n]$  are partitioned according to edge type. The two categories are interior edges, which are denoted by the subscript  $i$  in Equation (4), and dielectric boundary edges, which are denoted by the subscript  $d$  in Equation (4).  $[J_h]$  is a set of unknown complex scalar coefficients for the surface electric current densities on  $S_d$ .  $S_d$  is defined as  $S_2$  if the conductive body is not adjacent to the dielectric body; otherwise,  $S_d = S_2 - (S_2 \cap S_c)$ . Edges in  $[J_h]$  include dielectric boundary edges and junction edges (i.e. edges that are on both the conductor and dielectric boundary).  $[B_{dh}]$  is a square matrix if there are no junction edges in the problem.  $[g^{\text{int}}]$  is the source term, representing sources located within the FEM region. The elements of  $[A]$ ,  $[B_{dh}]$ , and  $[g^{\text{int}}]$  are given by,

$$A_{mn} = \int_{V_2} \left[ \frac{(\nabla \times \mathbf{w}_n(\mathbf{r})) \cdot (\nabla \times \mathbf{w}_m(\mathbf{r}))}{j\omega\mu_0\mu_r + j\omega\epsilon_0\epsilon_r \mathbf{w}_n(\mathbf{r}) \cdot \mathbf{w}_m(\mathbf{r})} \right] dV \quad (5)$$

$$B_{mn} = \int_{S_d} \mathbf{f}_n(\mathbf{r}) \cdot \mathbf{w}_m(\mathbf{r}) dS \quad (6)$$

$$g_m^{\text{int}} = - \int_{V_2} \left[ \mathbf{J}^{\text{int}} + \frac{1}{j\omega\mu_0\mu_r} (\nabla \times \mathbf{M}^{\text{int}}) \right] \cdot \mathbf{w}_m(\mathbf{r}) dV \quad (7)$$

where  $\mathbf{f}_n(\mathbf{r})$  is the surface basis function. Since the tangential  $\mathbf{E}$  field vanishes on  $S_c$ ,  $B_{mn}$  is evaluated over  $S_d$  rather than all of  $S_2$ . A detailed description of how to evaluate the terms in Equation (5) is provided in [11] (pp. 254-257). Appendix A describes how to evaluate Equation (6).  $[g_m^{\text{int}}]$  is the source term and is discussed in a later section.

The MoM surface integral equation evaluated by the MoM part of EMAPS is a form of the electric field integral equation (EFIE) [13],

$$\begin{aligned} \mathbf{E}^{\text{inc}}(\mathbf{r}) = & \frac{1}{2} \mathbf{E}(\mathbf{r}) + \oint_S \{ \mathbf{M}(\mathbf{r}') \times \nabla' G_0(\mathbf{r}, \mathbf{r}') \\ & + jk_0\eta_0 \mathbf{J}(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') \\ & - j\frac{\eta_0}{k_0} \nabla' \cdot \mathbf{J}(\mathbf{r}') \nabla' G_0(\mathbf{r}, \mathbf{r}') \} dS' \end{aligned} \quad (8)$$

where  $\mathbf{r} \in S$ ,  $S \equiv S_c \cup S_d$ ,  $\eta_0$  and  $k_0$  are the intrinsic impedance and wavenumber in free space, respectively, and

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \quad (9)$$

is the Green's function in free space. The integral term with a bar in Equation (8) denotes a principal-value integral. The singularity at  $\mathbf{r}=\mathbf{r}'$  is excluded. The equivalent surface electric and magnetic currents are defined as,

$$\mathbf{J}(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}), \quad \mathbf{r} \in S \quad (10)$$

$$\mathbf{M}(\mathbf{r}) = \mathbf{E}(\mathbf{r}) \times \hat{\mathbf{n}}, \quad \mathbf{r} \in S_d. \quad (11)$$

$\mathbf{M}(\mathbf{r})$  vanishes on  $S_c$ .  $\mathbf{J}(\mathbf{r})$  and  $\mathbf{M}(\mathbf{r})$  can be approximated by using the triangular basis function  $\mathbf{f}_n(\mathbf{r})$  proposed by Glisson [14]. On the surface  $S_d$ , the MoM basis function  $\mathbf{f}_n(\mathbf{r})$  and the FEM basis function  $\mathbf{w}_n(\mathbf{r})$  are related by,

$$\mathbf{w}_n(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{f}_n(\mathbf{r}), \quad \mathbf{r} \in S_d \quad (12)$$

$\mathbf{J}(\mathbf{r})$ ,  $\mathbf{M}(\mathbf{r})$  can be expanded as,

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N_s} J_n \mathbf{f}_n(\mathbf{r}) \quad (13)$$

$$\mathbf{M}(\mathbf{r}) = \sum_{n=1}^{N_d} E_n \mathbf{f}_n(\mathbf{r}) \quad (14)$$

where  $N_s$  is the total number of edges on the surface  $S$ , and  $N_d$  is the total number of edges on the surface  $S_d$ .  $\{J_n\}$  and  $\{E_n\}$  are unknown complex scalar coefficients.

The weighting functions chosen are  $\mathbf{f}_n(\mathbf{r})$ ,  $n=1, \dots, N_s$ . After multiplying by the weighting functions, Equation (8) can be discretized into Equation (15), which is a matrix equation. Edges on  $S_d$  and  $S_c$  are grouped together respectively.

$$\begin{bmatrix} C_{hh} & C_{hc} \\ C_{ch} & C_{cc} \end{bmatrix} \begin{bmatrix} J_h \\ J_c \end{bmatrix} = \begin{bmatrix} D_{hd} & 0 \\ D_{cd} & 0 \end{bmatrix} \begin{bmatrix} E_d \\ 0 \end{bmatrix} - \begin{bmatrix} F_h^i \\ F_c^i \end{bmatrix}. \quad (15)$$

The elements in matrices  $[C]$ ,  $[D]$  and  $[F^i]$  are given by,

$$\begin{aligned} C_{mn} = & -jk_0\eta_0 \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \left[ \int_{S_n} \mathbf{f}_n(\mathbf{r}') G_0(\mathbf{r}, \mathbf{r}') dS' \right] dS \\ & + j\frac{\eta_0}{k_0} \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \left[ \oint_{S_n} (\nabla' \cdot \mathbf{f}_n(\mathbf{r}')) \nabla' G_0(\mathbf{r}, \mathbf{r}') dS' \right] dS \end{aligned} \quad (16)$$

$$\begin{aligned} D_{mn} = & \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \\ & \left[ \oint_{S_n} \mathbf{f}_n(\mathbf{r}') \times \nabla' G_0(\mathbf{r}, \mathbf{r}') dS' + \frac{1}{2} \mathbf{w}_n(\mathbf{r}) \right] dS \end{aligned} \quad (17)$$

$$F_m^i = \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{E}^{\text{inc}}(\mathbf{r}) dS. \quad (18)$$

Details of how to evaluate Equation (16) are provided in [14]. Appendix B describes how to evaluate Equation (17). Equation (18) can be evaluated by using the Gaussian Quadrature technique.

From Equation (15),  $[J_h]$  can be expressed as,

$$\begin{bmatrix} 0 \\ J_h \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & C_{hh}^{-1} D'_{hd} \end{bmatrix} \begin{bmatrix} 0 \\ E_d \end{bmatrix} + \begin{bmatrix} 0 \\ C_{hh}^{-1} K \end{bmatrix} \quad (19)$$

where

$$C'_{hh} \equiv C_{hh} - C_{hc} C_{cc}^{-1} C_{ch} \quad (20)$$

$$D'_{hd} \equiv D_{hd} - C_{hc} C_{cc}^{-1} D_{cd} \quad (21)$$

$$K \equiv C_{hc} C_{cc}^{-1} F_c^i - F_h^i. \quad (22)$$



After combining Equations (4) and (19), the final matrix equation that must be solved is given by,

$$\left( [A] + \begin{bmatrix} 0 & 0 \\ 0 & -B_{dh} C_{hh}^{-1} D'_{hd} \end{bmatrix} \right) \begin{bmatrix} E_i \\ E_d \end{bmatrix} = [g^{int}] + \begin{bmatrix} 0 \\ B_{dh} C_{hh}^{-1} K \end{bmatrix} \quad (23)$$

where  $[A]$  is the FEM matrix in Equation (4), and  $B_{dh} C_{hh}^{-1} D'_{hd}$  is the coupled matrix from the MoM equation. The right-hand side in Equation (23) is the source term.

The far fields can be calculated using the equivalent surface currents  $\mathbf{J}$  and  $\mathbf{M}$  defined in Equations (10) and (11). The point of observation is denoted by  $(r, \theta, \phi)$  in spherical coordinates. Potential functions  $\mathbf{A}$  and  $\mathbf{F}$  can be approximated by [15],

$$\begin{aligned} \mathbf{A}(\mathbf{r}) &\equiv \frac{e^{-jk_0 r}}{4\pi r} \int_S \mathbf{J}(\mathbf{r}') e^{jk_0 r' \cos \psi} dS' \\ &= \frac{e^{-jk_0 r}}{4\pi r} \sum_n J_n \int_{S_n} \mathbf{f}_n(\mathbf{r}') e^{jk_0 r' \cos \psi} dS' \end{aligned} \quad (24)$$

$$\begin{aligned} \mathbf{F}(\mathbf{r}) &\equiv \frac{e^{-jk_0 r}}{4\pi r} \int_S \mathbf{M}(\mathbf{r}') e^{jk_0 r' \cos \psi} dS' \\ &= \frac{e^{-jk_0 r}}{4\pi r} \sum_n E_n \int_{S_n} \mathbf{f}_n(\mathbf{r}') e^{jk_0 r' \cos \psi} dS' \end{aligned} \quad (25)$$

where  $r' \cos \psi = x' \cos \phi \sin \theta + y' \sin \phi \sin \theta + z' \cos \theta$ . EMAP5 evaluates the integral terms in Equations (24) and (25) using the Gaussian Quadrature technique. The far fields are given by [15],

$$E_\theta = (-j k_0 \eta_0) (A_\theta + \frac{F_\theta}{\eta_0}) \quad (26)$$

$$E_\phi = (-j k_0 \eta_0) (A_\phi - \frac{F_\phi}{\eta_0}). \quad (27)$$

### III. SOURCE AND LOAD MODELING

EMAP5 can model voltage sources on metal patches, incident plane waves, and impressed current sources within the FEM region. A delta-gap voltage source can be defined anywhere on the MoM surface. It can be modeled as an incident E-field normal to edges coinciding with the source [9] (pp. 260-261). A delta-gap voltage source parallel to the x-axis can be expressed as,

$$\mathbf{E}^{inc} = V \delta(x - x_f) \hat{x} \quad (28)$$

where  $x_f$  is the location of the source and  $V$  is the voltage magnitude, and  $\delta(x)$  is the Dirac delta function. Suppose that edge  $m$  coincides with the source. The normal component of  $\mathbf{f}_m(\mathbf{r})$  across edge  $m$  is unity. Thus, the source term given by Equation (18) is,

$$F_m^i = V l_m \quad (29)$$

where  $l_m$  is the length of edge  $m$ .

Plane wave sources are also described using the forcing term given by Equation (18). Assume the incident wave can be expressed in a spherical coordinate system as,

$$\mathbf{E}^{inc} = (E_\theta \hat{\theta} + E_\phi \hat{\phi}) e^{-j \mathbf{k} \cdot \mathbf{r}} \quad (30)$$

where the propagation vector  $\mathbf{k}$  is,

$$\mathbf{k} = k(\sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}) \quad (31)$$

and  $(\theta, \phi)$  defines the angle of the propagation direction of the plane wave. EMAP5 calculates the E field of the plane wave on the surface of the structure, then evaluates the right-hand side of Equation (18) numerically.

EMAP5 uses current filaments on tetrahedron edges to model sources located within the FEM region [11] (pp. 324-325). A current source along the z-axis can be expressed as,

$$\mathbf{J}^{int} = I \delta(x - x_f) \delta(y - y_f) \hat{z} \quad (32)$$

where  $(x_f, y_f)$  specifies its position,  $I$  denotes the electric current magnitude, and  $\delta(x)$  is the Dirac delta function. The contribution to vector  $[g^{int}]$  in Equation (7) is simply,

$$g^{int} = I l \iint \hat{z} \cdot \{\mathbf{W}\} \delta(x - x_f) \delta(y - y_f) dx dy \quad (33)$$

where  $l$  is the edge length and  $\{\mathbf{W}\}$  is the weighting function. According to the expression in Equation (33), only edges coinciding with a current source have non-zero components in the  $[g^{int}]$  vector. After the E-fields along the current source edges are obtained, the voltage drop across the source can be calculated. Thus, the input impedance  $Z_{in}$  can be obtained. The desired fields under an excitation of any magnitude voltage  $V_{in}$  can be obtained by multiplying the factor  $V_{in}/Z_{in}$  to the results obtained when  $I$  is set to one Ampere.

EMAP5 models load impedances  $Z_L$  as dielectric posts on tetrahedron edges. These posts have a finite conductivity given by,

$$\sigma = \frac{l}{Z_L S} \quad (34)$$

where  $l$  is its length, and  $S$  is the cross sectional area. If the load is treated as a lumped element, its contribution to the finite element matrix is as follows [11] (pp. 324-325),

$$[A^e] = \frac{l^2}{Z_L} \iint \{W\} \{W\}^T \delta(x - x_L)(y - y_L) dx dy \quad (35)$$

where  $(x_L, y_L)$  is the position of the impedance load.

#### IV. MATRIX EQUATION SOLVER

EMAP5 uses the complex bi-conjugate gradient method (CBCG) to solve the final complex matrix equation, Equation (23). The major advantage of CBCG lies in its iterative nature and its ability to use the row-indexed method to store sparse matrices efficiently. The CBCG method was first introduced by Jacobs [16]. A brief outline of the algorithm is provided in the following section.

##### A. Complex Bi-Conjugate Gradient Method

Consider a complex matrix equation,

$$[M][x] = [b] \quad (36)$$

where  $[M]$  is an  $N \times N$  complex matrix, and  $[b]$  and  $[x]$  are order- $N$  complex column vectors. Both  $[M]$  and  $[b]$  are known while  $[x]$  is unknown. Four column vectors are defined as,

1. the residue,  $r$ ,
2. the bi-residue,  $\bar{r}$ ,
3. the direction vector,  $p$ ,
4. the bi-direction vector,  $\bar{p}$ .

These vectors are initialized as,

$$r_0 = p_0 = b \quad (37)$$

$$\bar{r}_0 = \bar{p}_0 = b^* \quad (38)$$

where  $*$  denotes the complex conjugate. The Hermitian conjugate of  $M$  is,

$$M^H \equiv (M^*)^T \quad (39)$$

where  $T$  denotes transpose. The inner product of two complex column vectors is defined as,

$$\langle p_1, p_2 \rangle \equiv (p_1^*)^T p_2 \quad (40)$$

The following steps are repeated  $N$  times at most. For  $k=0, \dots, N-1$ , calculate the step-length parameter,

$$\alpha_k = \frac{\langle \bar{r}_k, r_k \rangle}{\langle p_k, M p_k \rangle} \quad (41)$$

The new solution estimate then becomes,

$$x_{k+1} = x_k + \alpha_k p_k \quad (42)$$

The new residue and bi-residue are calculated as,

$$r_{k+1} = r_k - \alpha_k M p_k \quad (43)$$

$$\bar{r}_{k+1} = \bar{r}_k - \alpha_k^* M^H \bar{p}_k \quad (44)$$

The bi-conjugate coefficient is calculated using the following formula,

$$\beta_k = - \frac{\langle M^H \bar{p}_k, r_{k+1} \rangle}{\langle p_k, M p_k \rangle} \quad (45)$$

The new direction and bi-direction vectors are given by,

$$p_{k+1} = r_{k+1} + \beta_k p_k \quad (46)$$

$$\bar{p}_{k+1} = \bar{r}_{k+1} + \beta_k^* \bar{p}_k \quad (47)$$

This iteration is continued until a termination condition is satisfied,

$$\frac{\|r_k\|_2}{\|b\|_2} \leq \text{TOL} \quad (48)$$

where TOL is a number defining the tolerance. The best value of TOL depends on the problem for EMAP5. Typically,  $1 \times 10^{-3}$  is sufficient. The value of TOL is defined as a macro in the EMAP5 source code so it can be easily adjusted by the user.

##### B. Implementation

It is not necessary to explicitly add the two matrices on the left-hand side of Equation (23) in CBCG. The FEM matrix  $[A]$  is a sparse and symmetric matrix. The row-indexed scheme [17] is used to store it. The MoM coupled matrix is an asymmetric compact matrix. A full 2D array is used to store it. These two matrices are stored separately in order to make it is easier to obtain the Hermitian of the left-hand matrix. Since  $[A]$  is symmetric,

$$A^H = (A^T)^* = A^* \quad (49)$$

Thus, it is not necessary to search the non-zero elements column by column in  $[A]$  when Hermitian operations are present. Instead, the search is done row by row. The row-indexed scheme is very efficient in this case. While the coupled MoM matrix is not symmetric, exchanging its row index and column index will result in its transpose.

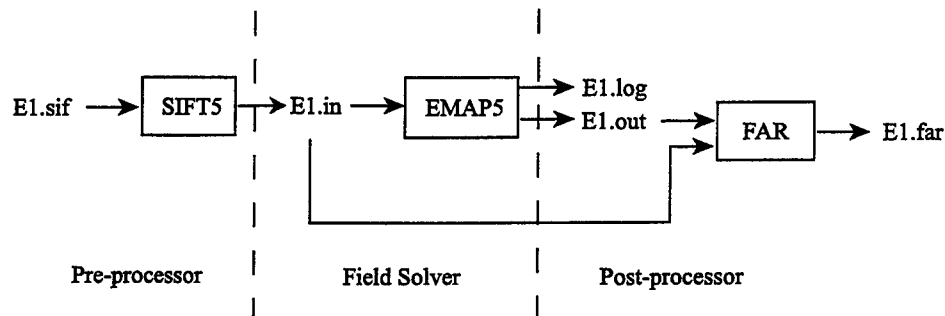


Figure 2. Components of the EMAP5 package.

### C. Accuracy and Convergence Problem

Although the CBCG solver conserves memory, it is an iterative method and it may have difficulty converging in some situations. Convergence problems may result from round-off errors. Usually the round-off error for single-precision data is  $1.19 \times 10^{-7}$  [17]. If the total number of unknown edges is 1,000, the average error due to round-off errors is

$$\sqrt{1,000} \times 1.19 \times 10^{-7} \approx 3.8 \times 10^{-6}.$$

In the worst case, the error is  $1,000 \times 1.19 \times 10^{-7} = 1.19 \times 10^{-4}$ . Because the total round-off error could be very large, it is impractical to set the TOL in Equation (48) to be on the order of  $10^{-4}$  if no special technique, such as pre-conditioning, is used to reduce the total round-off error. The round-off error of double-precision data is  $2.22 \times 10^{-16}$  [17]. If double precision is used for the matrix calculations, round-off errors will not reduce the effectiveness of CBCG considerably. In EMAP5, all data are stored in double precision; however, double precision numbers require twice as much as memory to store as single precision.

Another potential difficulty with the CBCG solver arises when the algorithm becomes unstable. This can happen when the denominator of the expressions for  $\alpha_k$  and  $\beta_k$  is close to zero. EMAP5 monitors the value of the denominator. If it becomes too small, EMAP5 will restart the algorithm using the current solution as the initial estimate as suggested by Jacobs [16].

## V. COMPONENTS OF EMAP5

EMAP5 is written in ANSI C. Thus, it can be compiled without modification on many computer platforms. As shown in Figure 2, the EMAP5 software package includes three major components: SIFT5, EMAP5 and FAR. EMAP5 was developed for research and educational use. It does not have a sophisticated mesh generator or graphical visualization tools. SIFT5 can be used to describe simple input geometries with a rectangular grid. It is also possible to use commercial

mesh generators to analyze more complex geometries of arbitrary shape. EMAP5 is the FEM/MoM field solver. FAR calculates far-field electric fields from the equivalent surface currents calculated by EMAP5.

Standard Input File Translator version 5 (SIFT5) is designed to generate input files for the field solver EMAP5. SIFT5 reads a text file in the SIFT format [18-20] and generates an input file for EMAP5. Users can describe the structure of interest by using eleven keywords. The physical geometry of the structure, source type and location, and the geometry of areas in which fields are to be printed by the solver, must be specified.

EMAP5 is the hybrid FEM/MoM field solver. It reads an input file generated by SIFT5. A .log file is generated to log running status and error messages. The log file was initially designed to debug the code, but users can also benefit from log messages. EMAP5 will print out fields within areas specified by the .sif file, to one or more output files. All equivalent surface currents  $\mathbf{J}$  and  $\mathbf{M}$  must be printed to a file if the user needs to calculate far fields.

FAR is a program used to calculate the far field radiation pattern. FAR needs two input files. One is the file generated by SIFT5 and the other contains the equivalent surface currents  $\mathbf{J}$  and  $\mathbf{M}$ . The FAR program will prompt users to input the observing distance  $R$  in wavelengths, the observing  $\theta$  interval in degrees, and the observing  $\phi$  interval in degrees.

## VI. STRUCTURE OF EMAP5

Figure 3 shows a flowchart for the field solver EMAP5. Descriptions of the functions used by EMAP5 are provided below,

- ReadInputFile()

reads an input file generated by SIFT5, and checks whether the input file is consistent (i.e. the input makes sense). If not, it aborts the program and reports problems with the input to the user.

- **ComputeBMatrix()**  
places a  $3 \times 3$  sub-matrix (corresponding to one observing triangular patch and one source triangular patch) into the global B matrix based on the local-to-global mapping.
- **ComputeCMatrix()**  
places a  $3 \times 3$  sub-matrix into the global C matrix based on the local-to-global mapping.
- **ComputeDMatrix()**  
places a  $3 \times 3$  sub-matrix into the global D matrix based on the local-to-global mapping.
- **ComputeCorrectionTerm()**  
uses B, C, and D matrices to compute the MoM coupling matrix. It also creates the necessary matrices required to compute the equivalent surface currents.
- **FEMMatrixCompute()**  
computes a  $6 \times 6$  matrix for each tetrahedron first, then assembles the global matrix based on the local-to-global mapping. The matrix is stored using the row-indexed method.
- **CreateRHSVector()**  
creates the RHS (right-hand side) vector of the final matrix equation.
- **ConjugateSolver()**  
uses the Complex Bi-Conjugate Gradient Method to solve the final matrix equation. For MoM-only analyses, this function can be replaced with an LU decomposition solver.
- **SurfaceFieldCompute()**  
computes the equivalent surface currents on the boundary.
- **PrintOutput()**  
outputs the values of fields in regions specified by the input file to a file specified by the input file. All equivalent surface electric and magnetic currents must be written to a file if users are interested in the far field pattern.

## VII. NUMERICAL RESULTS

This section describes four example geometries that can be solved using the EMAP5 software package. Each geometry is described in sufficient detail to allow others to duplicate the analysis. Results obtained from EMAP5 are compared to results obtained using other well-established codes, measurements, or analytical results.

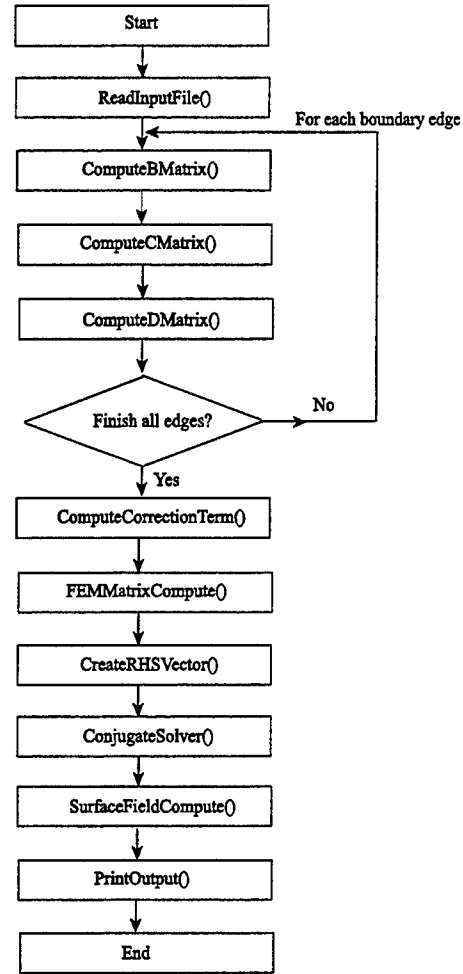


Figure 3. EMAP5 flowchart.

The first example analyzes the electromagnetic scattering from a square plate over a dielectric slab. As shown in Figure 4, the configuration consists of a square plate of dimension  $0.75\lambda \times 0.75\lambda$ , and a dielectric slab of  $\epsilon_r = 2.0$ , and dimensions of  $0.4\lambda \times 0.4\lambda \times 0.2\lambda$ . The plate is symmetrically placed above the slab. The distance between the plate and the dielectric slab is  $0.15\lambda$ . The system is illuminated by an x-polarized, +z traveling plane wave. In EMAP5's hybrid solution, the dielectric is divided into 320 tetrahedra. The surface of the dielectric and the metal plate are divided into 392 triangle patches. Figure 5 shows the normalized far field pattern calculated by EMAP5. For comparison, the results obtained by Arvas *et al.* [21] have been plotted on the same scale. The authors in [21] used a method of moments technique, which is based on a combined field integral equation formulation. As is evident from the figure, the two solutions compare very well.

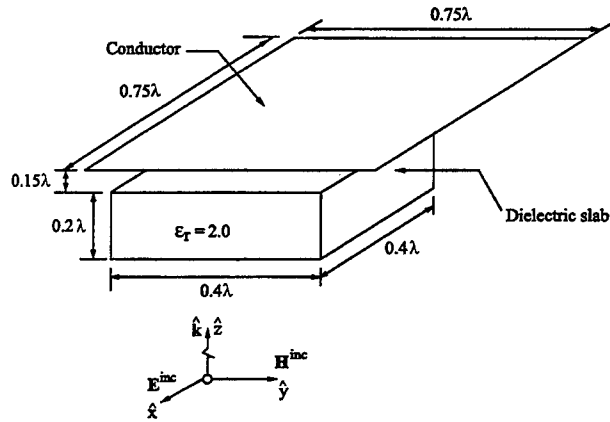


Figure 4. A conducting plate over a dielectric slab.

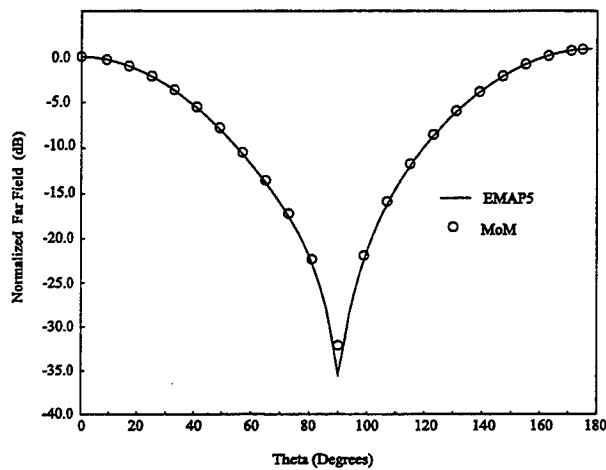


Figure 5. The normalized far field pattern for the configuration in Figure 4.

The second example demonstrates the application of the EMAP5 code in modeling microstrip geometries. The geometry of the structure is shown in Figure 6. The board is made of a dielectric with  $\epsilon_r=4.0$ . The trace is excited by a source at one end, and is terminated by a resistor at the other end. Microstrip lines support quasi-TEM fields [22] (pp. 160-161) and can therefore be viewed as transmission lines. The input impedance  $Z_{in}$  of a transmission line is given by,

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_L \tan(\beta l)} \quad (50)$$

where  $Z_L$  is the load impedance,  $\beta$  is the propagation constant;  $l$  is the length of the transmission line. To determine the characteristic impedance  $Z_0$  of the transmission line, one can find the input impedance when the load side is shorted or open, respectively. When the load side is shorted, the input impedance is given by,

$$(Z_{in})_{short} = j Z_0 \tan(\beta l). \quad (51)$$

When the load side is open, the input impedance is given by,

$$(Z_{in})_{open} = -j \frac{Z_0}{\tan(\beta l)}. \quad (52)$$

Thus, the characteristic impedance is given by,

$$Z_0 = \sqrt{(Z_{in})_{short} (Z_{in})_{open}}. \quad (53)$$

In EMAP5's hybrid solution, the dielectric volume is divided into 900 tetrahedra. The surface is divided into 828 triangles. Table 1 lists  $(Z_{in})_{open}$ ,  $(Z_{in})_{short}$  and  $Z_0$  obtained using EMAP5 at three different frequencies. The results in Table 1 are consistent and  $Z_0 = 56.4$  Ohms at each frequency. Figure 7 shows the input impedance at the source end obtained by EMSIM [23] when the trace is terminated with a  $56.4 \Omega$  resistor. It is evident that the microstrip line is almost perfectly matched confirming EMAP5's calculation of the characteristic impedance.

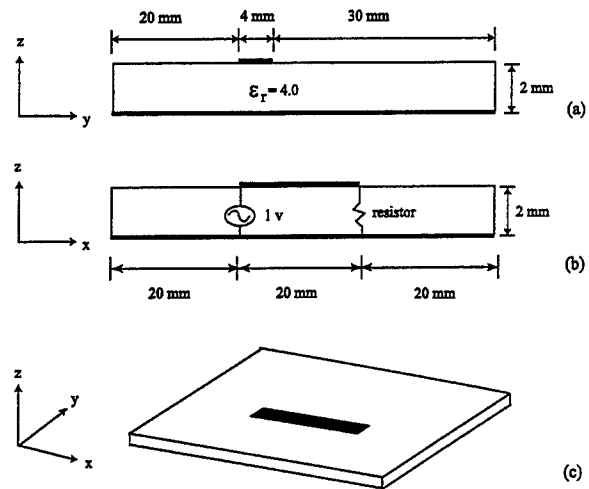


Figure 6. A microstrip geometry. (a) y-z plane view, (b) x-z plane view, and (c) 3D view.

Table 1. Impedances obtained using EMAP5.

Source Frequency (MHz)	$(Z_{in})_{open}$ (Ohms)	$(Z_{in})_{short}$ (Ohms)	$Z_0$ (Ohms)
300	-j187	+j17	56.4
500	-j106	+j30	56.4
700	-j69	+j46	56.3

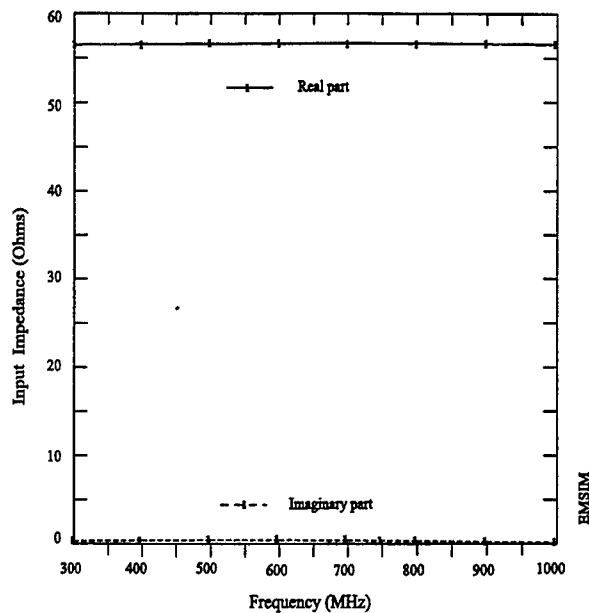


Figure 7. The input impedance obtained using EMSIM with a 56-Ohm load resistor.

The third example simulates the coupling between an active trace and an off-board trace on a printed circuit board. As shown in Figure 8, the active trace is excited by a 1-volt, 600 MHz source at one end, and is terminated by a 50-Ohm resistor at the other end. The off-board trace runs parallel to the active trace and extends well off the board. In EMAP5's hybrid solution, the dielectric volume is divided into 900 tetrahedra. The surface is divided into 876 triangles. For the first test case, there is no dielectric material between the traces and the ground. Therefore, the model contains only perfect electrical conductor (PEC) and can be analyzed using only the MoM portion of EMAP5. Figure 9 shows the current on the off-board trace obtained by EMAP5 compared with results obtained by EMSIM. For the second test case, the dielectric constant of the slab is set to 4.0. Figure 10 shows the results obtained by EMAP5 compared with results obtained using EMSIM. For both cases, good agreement is obtained between EMAP5 and EMSIM.

The fourth example models a printed circuit board power bus structure. As shown in Figure 11, the top and the bottom planes of the board are PEC. The dielectric between the top and bottom planes has a relative permittivity of 4.3(1-j0.02). Two test ports are identified at the locations shown in Figure 11. In EMAP5's hybrid solution, the dielectric volume is divided into 1,500 tetrahedra. The surface of the dielectric and the metal plates are divided into 1,340 triangle patches. The model is verified using measurements of an actual printed circuit board with the dimensions shown in Figure 11. Two 85-mil semi-

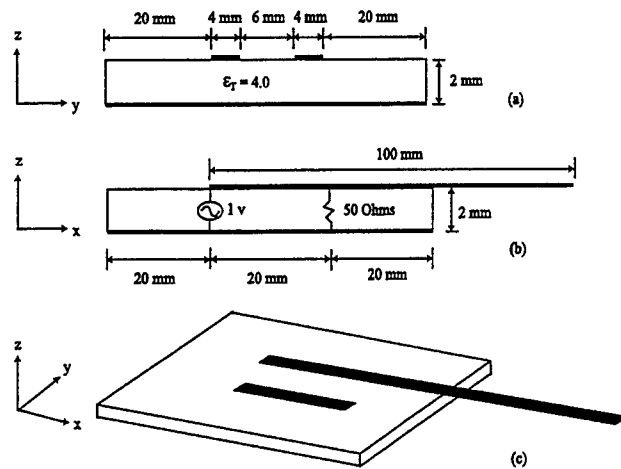


Figure 8. A grounded printed circuit board with active and passive traces. (a) y-z plane view, (b) x-z plane view, and (c) 3D view.

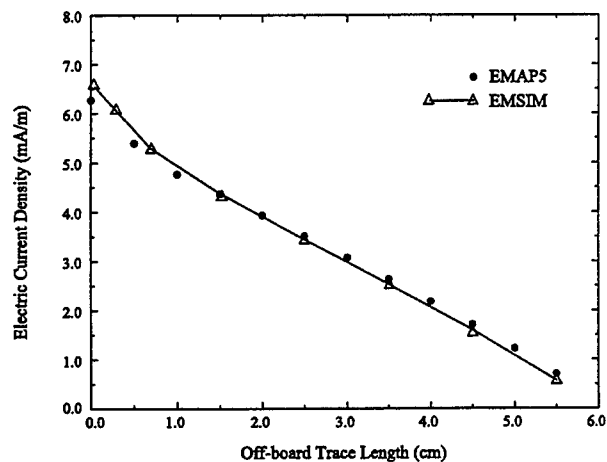


Figure 9. Current distribution on the off-board trace ( $\epsilon_r=1.0$ ).

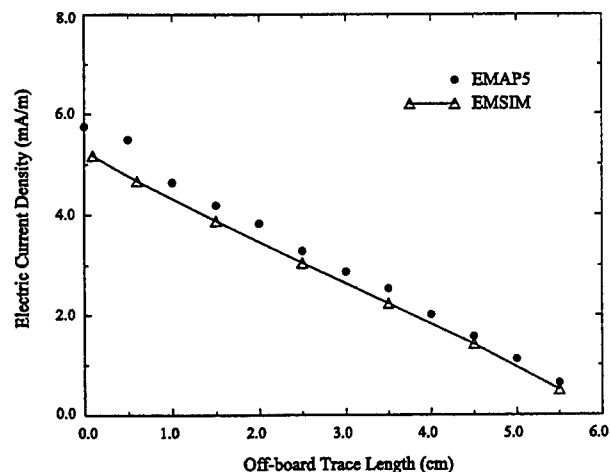


Figure 10. Current distribution on the off-board trace ( $\epsilon_r=4.0$ ).

rigid coaxial probes are attached at the port locations. A network analyzer is used to measure the scattering parameters of the two-port system up to 2.0 GHz. The measured and modeled  $|S_{11}|$  and  $|S_{21}|$  results are plotted in Figure 12 and Figure 13, respectively.

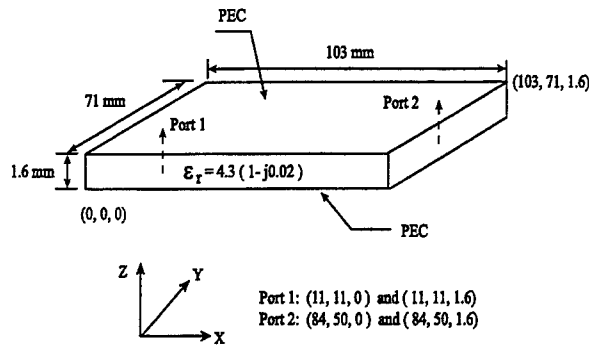


Figure 11. A two-port power bus structure.

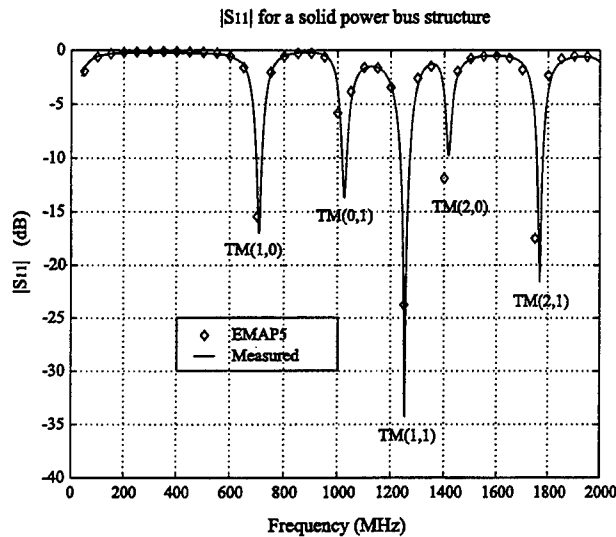


Figure 12. The measured and calculated  $|S_{11}|$  for the power bus structure.

The power bus structure can be analyzed as a cavity with two PEC and four perfect magnetic conductor (PMC) walls. The cutoff frequencies are given as follows (for lossless case and  $\mu_r = 1.0$ ) [22],

$$f_c = \frac{c}{2\pi\sqrt{\epsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad m, n = 0, 1, 2, \dots \quad (54)$$

where  $a$  and  $b$  are the length and width of the cavity, respectively;  $m$  and  $n$  are the mode indices;  $c$  is the light speed in free space;  $\epsilon_r$  is the relative permittivity of the material in the cavity. For this power bus structure, only  $TM_z$  modes are excited.

Table 2 lists the calculated cutoff frequencies of the first five  $TM_z$  modes. As shown in Figure 12, the  $|S_{11}|$  response dips at these resonant frequencies. At resonance, more power is delivered to the board hence more power is transmitted to Port 2 as shown in Figure 13. The measurements, numerical results and theoretical analysis agree very well.

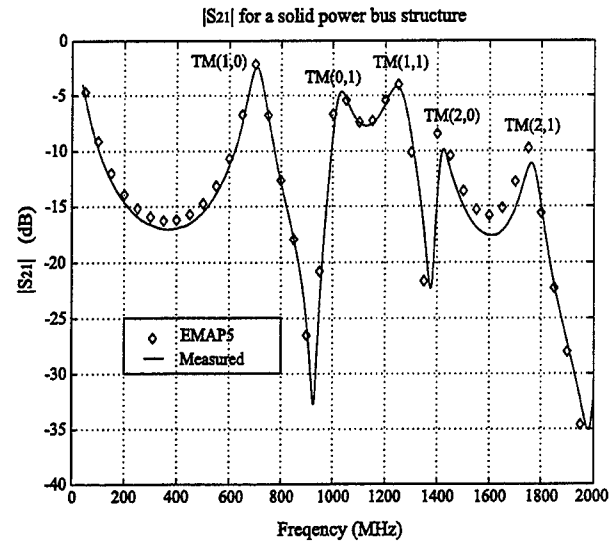


Figure 13. The measured and calculated  $|S_{21}|$  for the power bus structure.

Table 2. The  $TM_z$  mode cutoff frequencies of the power bus structure.

Mode	$TM_z$ (1,0)	$TM_z$ (0,1)	$TM_z$ (1,1)	$TM_z$ (2,0)	$TM_z$ (2,1)
$f_c$ (GHz)	0.702	1.019	1.237	1.405	1.735

## VIII. SUMMARY

The EMAP5 hybrid FEM/MoM code allows users to analyze geometries with large PEC surfaces and lossy dielectrics. Unlike codes that simply use the MoM surface as an absorbing boundary for the FEM region, EMAP5 can model PEC surfaces inside, outside and adjacent to the boundary between FEM and MoM regions. This permits a variety of interesting geometries to be modeled efficiently. The SIFT5 input translation code allows users to input simple 3D geometries to EMAP5 without a graphical user interface. Four sample geometries were analyzed that demonstrate the types of problems that can be analyzed

using SIFT5 and EMAP5. Good agreement was achieved between EMAP5 results and other well-established codes, analytical results, and measurements.

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### Appendix A. Evaluation of B Matrix

The elements in B are given by,

$$B_{mn} = \int_{S_m} \mathbf{w}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}) dS. \quad (\text{A-1})$$

When edge  $m$  and  $n$  do not lie on a common triangle,  $B_{mn}$  is zero. In addition,  $B_{mn} = 0$  when  $m=n$ , because

$$\mathbf{w}_n(\mathbf{r}) = \hat{\mathbf{n}} \times \mathbf{f}_n(\mathbf{r}). \quad (\text{A-2})$$

If edge  $m$  and  $n$  lie on a common triangle  $T^q$ ,  $B_{mn}$  is given by,

$$\begin{aligned} B_{mn} &= \int_{S_m} \mathbf{w}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}) dS = \int_{S_m} [\hat{\mathbf{n}} \times \mathbf{f}_m(\mathbf{r})] \cdot \mathbf{f}_n(\mathbf{r}) dS \\ &= \int_{S_m} \hat{\mathbf{n}} \cdot [\mathbf{f}_m(\mathbf{r}) \times \mathbf{f}_n(\mathbf{r})] dS = \hat{\mathbf{n}} \cdot \int_{S_m} [\mathbf{f}_m(\mathbf{r}) \times \mathbf{f}_n(\mathbf{r})] dS \\ &= \left( \frac{\pm l_m}{2A^q} \right) \left( \frac{\pm l_n}{2A^q} \right) \hat{\mathbf{n}} \cdot \int_{S_m} [(\mathbf{r} - \mathbf{r}_m) \times (\mathbf{r} - \mathbf{r}_n)] dS \\ &= \left( \frac{\pm l_m}{2A^q} \right) \left( \frac{\pm l_n}{2A^q} \right) \hat{\mathbf{n}} \cdot \int_{S_m} [\mathbf{r}_m \times \mathbf{r}_n + (\mathbf{r}_n - \mathbf{r}_m) \times \mathbf{r}] dS. \quad (\text{A-3}) \end{aligned}$$

The  $\pm$  signs before  $l_m$  and  $l_n$  refer to the edge directions of edge  $m$  and edge  $n$ , respectively. The integral term in Equation (A-3) can be evaluated using the one-point Gaussian Quadrature technique,

$$B_{mn} = \frac{(\pm l_m)(\pm l_n)}{4A^q} \hat{\mathbf{n}} \cdot [\mathbf{r}_m \times \mathbf{r}_n + (\mathbf{r}_n - \mathbf{r}_m) \times \mathbf{r}^{\text{cp}}] \quad (\text{A-4})$$

where  $\mathbf{r}^{\text{cp}}$  is the centroid of triangle  $T^q$ .

### Appendix B. Evaluation of Matrix D

The elements in matrix D are given by,

$$\begin{aligned} D_{mn} &= \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \left[ \int_{S_n} \mathbf{f}_n(\mathbf{r}') \times \nabla' G_0(\mathbf{r}, \mathbf{r}') dS' + \frac{1}{2} \mathbf{w}_n(\mathbf{r}) \right] dS \\ &= -\frac{1}{2} B_{mn} + D'_{mn} \quad (\text{since } B_{mn} = -B_{nm}) \quad (\text{B-1}) \end{aligned}$$

where  $D'_{mn}$  is defined as follows,

$$\begin{aligned} D'_{mn} &= \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \left[ \int_{S_n} \mathbf{f}_n(\mathbf{r}') \times \nabla' G_0(\mathbf{r}, \mathbf{r}') dS' \right] dS \\ &= \left( \frac{\pm l_m}{2A^m} \right) \left( \frac{\pm l_n}{2A^n} \right) \int_{S_m} (\mathbf{r} - \mathbf{r}_m) \\ &\quad \cdot \left[ \int_{S_n} (\mathbf{r}' - \mathbf{r}'_n) \times \nabla' G_0(\mathbf{r}, \mathbf{r}') dS' \right] dS \quad (\text{B-2}) \end{aligned}$$

$$\nabla' G_0(\mathbf{r}, \mathbf{r}') = \frac{1 + j k_0 R}{4\pi R^3} e^{-jk_0 R} \mathbf{R} \quad (\text{B-3})$$

where  $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ ,  $R = |\mathbf{R}|$ . The  $\pm$  signs before  $l_m$  and  $l_n$  refer to the edge directions of edge  $m$  and edge  $n$ , respectively. Because the EFIE given by Equation (8) already excludes the singularity at  $\mathbf{r} = \mathbf{r}'$ ,  $D'_{mn}$  can be evaluated directly using the seven-point Gaussian Quadrature technique.

# A Hybrid MoM/FEM Technique for Scattering from a Complex BOR with Appendages

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## Abstract

A hybrid technique is developed to allow the scattering from small appendages to be approximately combined with the scattering from a large body of revolution (BOR). The hybrid technique thus enables the rotational symmetry of the large BOR to be exploited to solve the scattering problem with much less computational complexity than a fully three-dimensional (3-D) solution. The technique combines the finite element method (FEM) for BOR scattering with the method of moments (MoM) for small appendages. The hybrid formulation is discussed in detail, including an approximate Green's function which permits the decoupling of the solutions from the FEM and the MoM. Numerical examples are given to show the applicability and accuracy of the hybrid technique.

## 2 Formulation

As mentioned, the two methods to be hybridized are the FEM for BOR scattering and the MoM for small appendages. The FEM for BOR scattering makes use of edge-based vector basis functions to expand the transverse field components and node-based scalar basis functions to expand the angular component. This mixed edge-node formulation eliminates the problem of spurious modes [4], and it directly computes all components of the electric field. The coupled azimuth potential formulation, which computes the angular component of the electric and magnetic fields, is less accurate than the mixed edge-node formulation [5]. Mesh truncation for the FEM is accomplished using cylindrical perfectly matched layer (PML), which is efficient and accurate. PML does not alter the sparsity of the FEM matrix, and it avoids the wasted computation required by a spherical mesh boundary for an elongated geometry. Finally, the FEM equations are solved using band matrix decomposition techniques. Validation examples showing the efficiency and accuracy of the method are found in [4].

## 1 Introduction

The symmetry present in body of revolution (BOR) electromagnetic scattering problems permits an efficient numerical solution using a two-dimensional (2-D) technique [1]–[5]. However, in many practical problems, the rotational symmetry is broken by the presence of small appendages (see Fig. 1). Thus, a three-dimensional (3-D) computational method is required to rigorously compute the electromagnetic scattering. Because of the increased computational complexity of a 3-D method, a hybrid method is developed to allow the scattering from small appendages to be approximately combined with the scattering from a large BOR in a manner similar to other hybrid techniques [6]–[9]. The rotational symmetry can then be exploited in the computation of the scattering from the large BOR. The hybrid method makes use of the finite element method (FEM) for BOR scattering as described in [4], and the method of moments (MoM) for small appendages as described in [8]. The remainder of this paper discusses the hybridization of the two methods. The formulation is presented in Section 2, numerical results are given in Section 3, and concluding remarks are found in Section 4.

The MoM used with the hybrid method is straightforward and makes use of the well-known Rao-Wilton-Glisson (RWG) basis functions. The MoM matrix equations encountered are typically of low dimension and are thus solved using simple LU-decomposition techniques. If necessary, fast multipole techniques can be employed for larger problems [10].

The hybrid formulation is best understood by first considering the computation of the scattering from a BOR with appendages (see Fig. 1) entirely by the MoM. Assuming that the entire target consists of perfect conductors, the application of the MoM uses the integral equation

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) - jk_0\eta_0 \iint_S \mathbf{G}_0(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' \quad (1)$$

where  $k_0 = \omega\sqrt{\mu_0\epsilon_0}$  is the free-space wavenumber,  $\eta_0 = \sqrt{\mu_0/\epsilon_0}$  is the impedance of free space,  $\mathbf{G}_0$  is the free-space dyadic Green's function,  $\mathbf{E}^i$  is a known incident electric field,  $\mathbf{J}$  is the unknown current on the surface of the target, and  $S$  is the surface of the entire target. When using the MoM,  $\mathbf{J}$

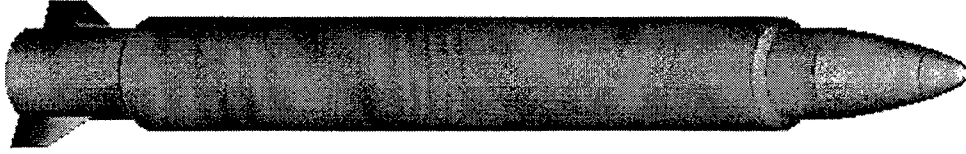


Figure 1: Example of a large BOR with small appendages.

is found by discretizing the surface  $S$  and applying the appropriate boundary condition on the discretized surface. The scattered field is then found from

$$\mathbf{E}^s(\mathbf{r}) = -jk_0\eta_0 \iint_S \mathbf{G}_0(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS'. \quad (2)$$

This 3-D method generates a dense matrix equation; therefore, if the geometry is large, the method is computationally very intensive. Further, if the BOR contains inhomogeneous materials, the surface integral in Eq. (1) must be replaced with a volume integral, further increasing the computational complexity.

Because of the unfavorable computational complexity in solving Eq. (1) with the MoM, an alternate, hybrid method is sought. In the hybrid method, the MoM is applied to the small appendages only. Thus, Eq. (1) becomes

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\text{BOR}}^i(\mathbf{r}) - jk_0\eta_0 \iint_{S_{\text{App}}} \mathbf{G}_{\text{BOR}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' \quad (3)$$

where  $\mathbf{E}_{\text{BOR}}^i$  represents the incident field on the appendages in the presence of the BOR,  $S_{\text{App}}$  represents the surface of the appendages, and  $\mathbf{G}_{\text{BOR}}$  represents the dyadic Green's function in the presence of the BOR. The solution of Eq. (3) requires the discretization of  $S_{\text{App}}$  rather than  $S$  and thus is much less computationally intensive. The incident field  $\mathbf{E}_{\text{BOR}}^i$  is calculated using the FEM [4], but difficulty is still encountered in solving Eq. (3) with the MoM because the Green's function  $\mathbf{G}_{\text{BOR}}$  is, in general, unknown. The unknown Green's function is needed not only for the solution of Eq. (3), but also to compute the scattered electric field from

$$\mathbf{E}^s(\mathbf{r}) = -jk_0\eta_0 \iint_{S_{\text{App}}} \mathbf{G}_{\text{BOR}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS'. \quad (4)$$

The difficulty posed by the unknown Green's function  $\mathbf{G}_{\text{BOR}}$  is alleviated in two ways. An approximate Green's function is used for the solution of Eq. (3), and an alternate method based on the reciprocity theorem is used to compute  $\mathbf{E}^s$ .

There are four steps to using the hybrid method. First, the FEM described in [4] is used to compute scattering from the large BOR alone. Then, the result of the first step is used to compute  $\mathbf{E}_{\text{BOR}}^i$ , the incident field on the appendages in the presence of the BOR. Next, the MoM with an approximate Green's function is used to solve Eq. (3) for  $\mathbf{J}$ , the current on the appendages. Note that in the usual MoM analysis with RWG basis functions, the unknowns are placed on

edges which are shared by two facets in the mesh. In the hybrid method, unknowns must also be placed on the boundary edges connecting the appendages to the large BOR to account for current flow from the appendages to the large BOR [8]. Finally, the scattered field generated by  $\mathbf{J}$  is evaluated using reciprocity and added to the scattered field from the BOR, which is found in the first step. The result is an approximation to the scattered field from the entire structure. In the remainder of this section, the construction of the approximate Green's function and the computation of the scattered field by reciprocity are each discussed in turn.

## 2.1 Approximate Green's function

The approximate Green's function used to solve Eq. (3) is developed by approximating the BOR on which the appendages reside as a long cylinder. The approximate Green's function must model the significant field interactions between points on the appendages but may neglect other, less significant interactions. The interactions are classified into four types. The first type of interaction is the direct path interaction between two points, and examples of this type of interaction are illustrated in Fig. 2a. The second type of interaction involves a reflection by the large BOR, and examples are illustrated in Fig. 2b. The third interaction type is the surface wave interaction which is illustrated in Fig. 2c. All other interactions are classified as the fourth type. Most of these are complex interactions which are not modeled by the cylinder approximation to the BOR, and most are negligible.

If the line of sight between two points on the appendages is unobstructed by the cylinder approximation to the large BOR, the first two interaction types can be computed using the half-space Green's function. This is done by finding the reflection point on the cylinder and computing the tangent plane to the cylinder at that point. If the line of sight between the two appendage points is obstructed by the cylinder, both of the first two interactions are zero. For many problems, modeling the first two interactions produces acceptable results. If better accuracy is desired, the third type of interaction can be computed using the geometrical theory of diffraction (GTD) [11]. Also, if the appendages are near the end of the BOR, it may be necessary to compute an interaction based on edge diffraction from the nearby end. Note that the effect of neglecting some of the interactions is to neglect the

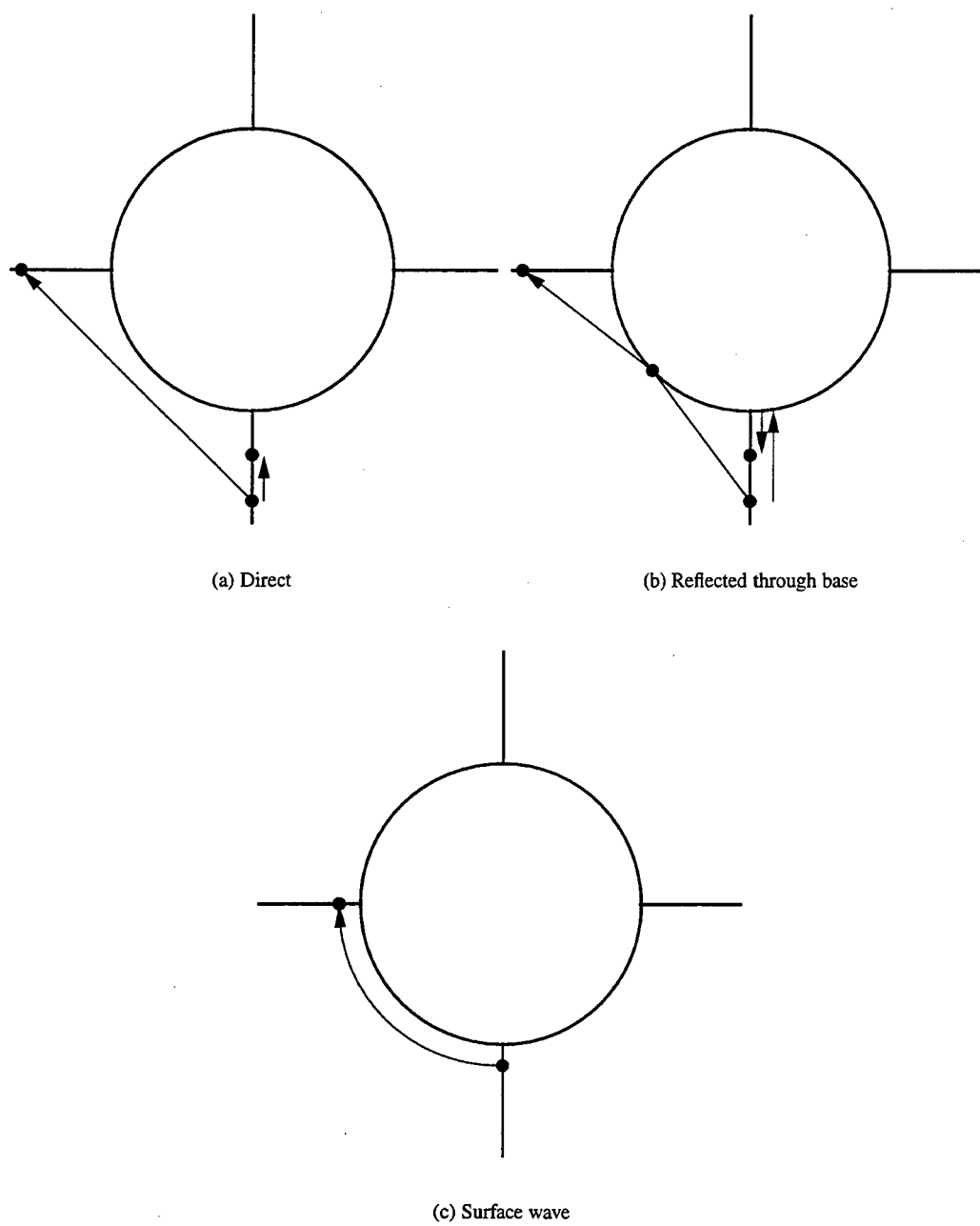


Figure 2: Interactions between appendage points.

corresponding field which is scattered by the appendages, reflected or diffracted back to the appendages, and scattered by the appendages again [6]–[9].

In all of the examples presented in this paper, the approximate Green's function models the first two interaction types (Fig. 2a-b) and neglects all other interactions. To compute values of the approximate Green's function, the geometry of the protrusions is first examined, and the radius of the infinite cylinder used to approximate the large BOR is found from the points where the protrusions attach to the BOR. Given any two points on the protrusions, the value of the Green's function is then computed in a four step process. First, the direct (line of sight) path between the two points is computed. If this path is blocked by the infinite cylinder, there is no line of sight interaction. Next, the interaction involving a reflection by the infinite cylinder is considered. The reflection point is found using Snell's law. Note that it can be shown that if there is no direct interaction, there is also no reflected interaction. Thus, if there is no direct path, the value of the approximate Green's function between the two points is zero. If there is a direct path, the third step is the application of a coordinate translation and rotation such that in the transformed space, the reflection point is on the  $z = 0$  plane and the normal to the cylinder at the reflection point is in the  $\hat{z}$  direction. Finally, the half-space Green's function is applied in the transformed space to compute the value of the approximate Green's function between the two given points.

## 2.2 Computation of scattered field

While the use of the approximate Green's function allows Eq. (3) to be solved for  $\mathbf{J}$ , the approximation is not accurate when computing  $\mathbf{E}^s$  from Eq. (4). Therefore, an alternate method of computing  $\mathbf{E}^s$  is used. The alternate method is based on the reciprocity theorem. Consider a short dipole, located at point  $\mathbf{r}$  and oriented in the  $\hat{u}$  direction. From the reciprocity theorem,

$$\mathbf{E}^s(\mathbf{r}) \cdot \hat{u} = -jk_0\eta_0 \frac{e^{-jk_0r}}{4\pi r} \iint_{S_{\text{App}}} \mathbf{E}_{\text{BOR}}^r(\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}') dS' \quad (5)$$

where  $\mathbf{E}_{\text{BOR}}^r$  is the field radiated by the dipole in the presence of the large BOR. Recall that this field can be computed by the FEM. In fact, when backscattering is being computed,  $\mathbf{E}_{\text{BOR}}^r$  is the same as  $\mathbf{E}_{\text{BOR}}^i$  that is used in Eq. (3). Thus, all components needed to compute  $\mathbf{E}^s$  using Eq. (5) are known.

## 3 Numerical Results

Several numerical results are presented to show the validity and capability of the hybrid technique. In all of the results presented, the direct and reflected interactions are modeled by the approximate Green's function, and all other interactions are neglected.

First, the validity of the technique is tested by computing the scattering from metallic cylinders with capped ends and one, two, or four wings. The scattering is compared with computations from the Fast Illinois Solver Code (FISC) [10], which is an MoM program that uses a multilevel fast multipole algorithm to speed up the matrix solution. The cylinders considered have a radius of  $1.25\lambda$  and height of  $5\lambda$ , where  $\lambda$  is the free-space electromagnetic wavelength. The cap at one end of the cylinder is pointed, while the cap at the other end is rounded. The attached wings are  $1\lambda$  by  $0.5\lambda$  by  $0.025\lambda$ .

The monostatic RCS as a function of azimuth angle from the capped cylinder with one wing is shown in Fig. 3. The agreement between FISC and the hybrid method is very good. The scattering from the cylinder without the wing is flat as a function of azimuth angle, so it can be seen that the wing has added approximately 5 dB peak-to-peak swing to the RCS in the VV-polarized case and about 4 dB peak-to-peak swing in the HH-polarized case. Although adding a second wing presents the possibility of more complex interactions, the results shown in Fig. 4 continue to show good agreement between FISC and the hybrid method. The peak-to-peak variation in the RCS is about 4.5 dB in the VV-polarized case and almost 6 dB in the HH-polarized case when two wings are present. The scattering from the cylinder with four wings is shown in Fig. 5, where the agreement between FISC and the hybrid method remains very good. With four wings present, the peak-to-peak swing in the RCS is about 5 dB in the VV-polarized case and about 6 dB in the HH-polarized case.

To further illustrate the capability of the hybrid method, two more computed results are presented. Both of these results involve 1-GHz scattering from a missile with appendages. The missile is 12.5 m ( $41.7\lambda$ ) long and has a radius of  $0.625$  m ( $2.1\lambda$ ). For the first case, a 3-cm ( $0.1\lambda$ ) by 3-cm ( $0.1\lambda$ ) by 8.125-m ( $27.7\lambda$ ) ridge is located on the missile at azimuth angle  $0^\circ$ , and computed results for an azimuth scan and for an elevation scan are presented in Fig. 6. Note that in the azimuth scan, the ridge causes the scattering to vary over a 3.5-dB range in the VV-polarized case and over a 3-dB range in the HH-polarized case while the missile alone, because of its rotational symmetry, has a constant RCS as a function of azimuth angle. In the second case, two fins are located on the missile at azimuth angles  $90^\circ$  and  $-90^\circ$ . The fins are trapezoidal in shape with a height of  $0.375$  m ( $1.25\lambda$ ), bases of  $1$  m ( $3.33\lambda$ ) and  $0.5$  m ( $1.67\lambda$ ), and a thickness of  $0.01$  m ( $0.03\lambda$ ). The scattering is shown both for an azimuth cut and for an elevation cut in Fig. 7. In the azimuth cut, the scattering has changed from flat for the missile alone to a function with a 3 dB peak-to-peak variation in the VV-polarized case and a 2 dB peak-to-peak variation in the HH-polarized case.

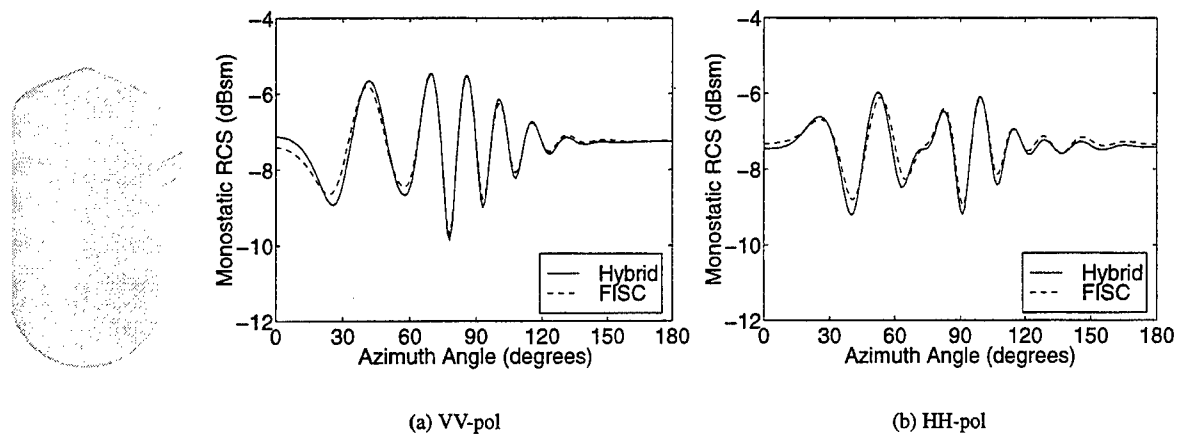


Figure 3: RCS of a metallic capped cylinder with a wing. The cylinder has a radius of  $1.25\lambda$  and height of  $5\lambda$ , and the wing is  $1\lambda$  by  $0.5\lambda$  by  $0.025\lambda$ .

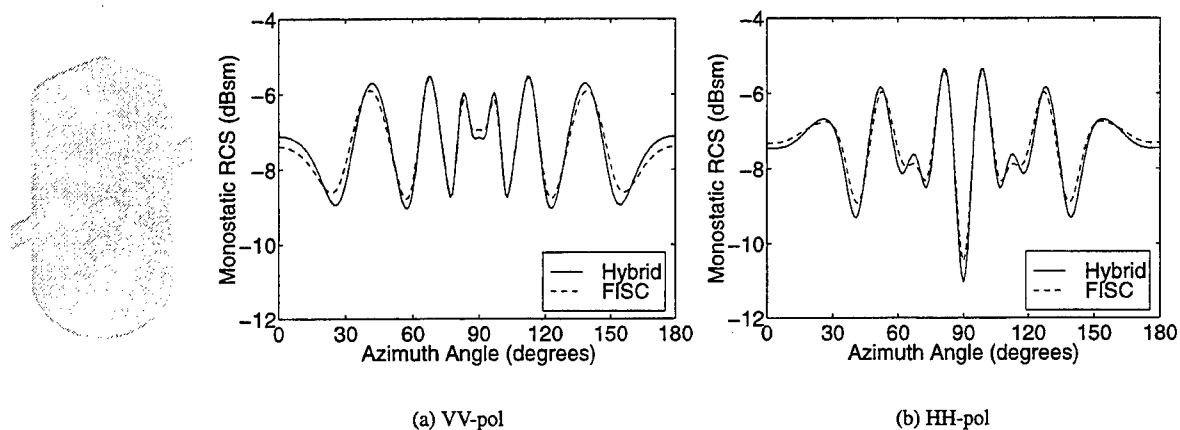


Figure 4: RCS of a metallic capped cylinder with two wings. The cylinder has a radius of  $1.25\lambda$  and height of  $5\lambda$ , and the wings are  $1\lambda$  by  $0.5\lambda$  by  $0.025\lambda$ .

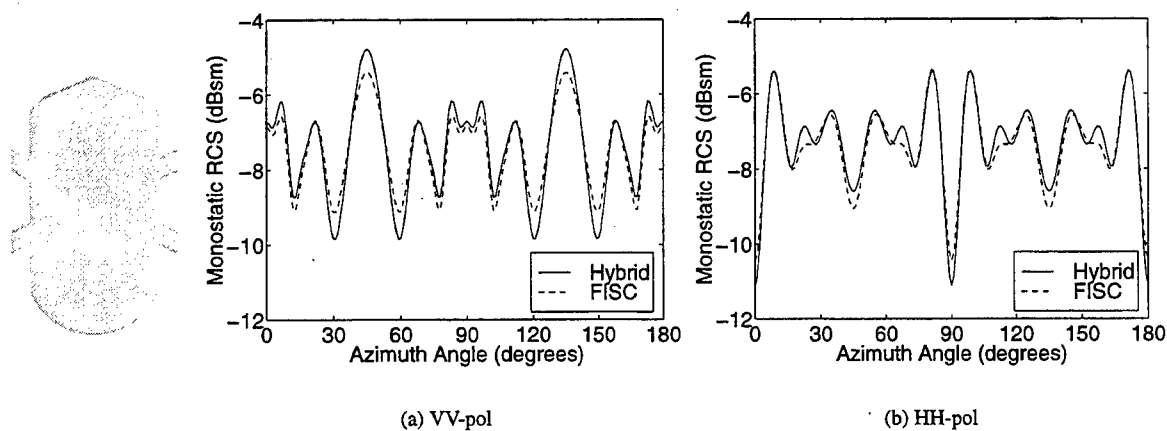


Figure 5: RCS of a metallic capped cylinder with four wings. The cylinder has a radius of  $1.25\lambda$  and height of  $5\lambda$ , and the wings are  $1\lambda$  by  $0.5\lambda$  by  $0.025\lambda$ .

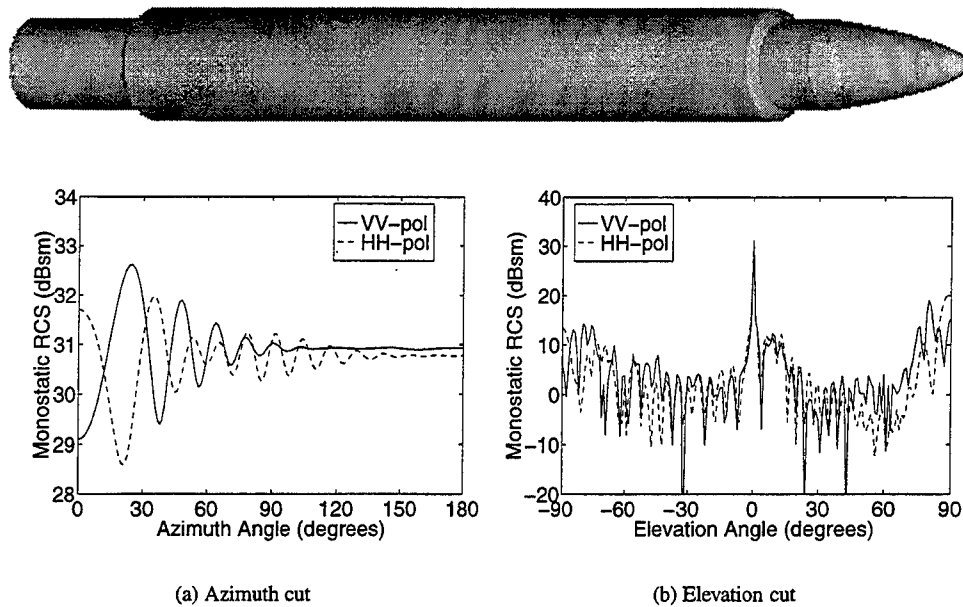


Figure 6: RCS of a missile with a ridge at 1 GHz. The missile is 12.5 m ( $41.7\lambda$ ) long and has a radius of 0.625 m ( $2.1\lambda$ ). The ridge is 3 cm ( $0.1\lambda$ ) by 3 cm ( $0.1\lambda$ ) by 8.125 m ( $27.7\lambda$ ) and located at azimuth angle  $0^\circ$ .

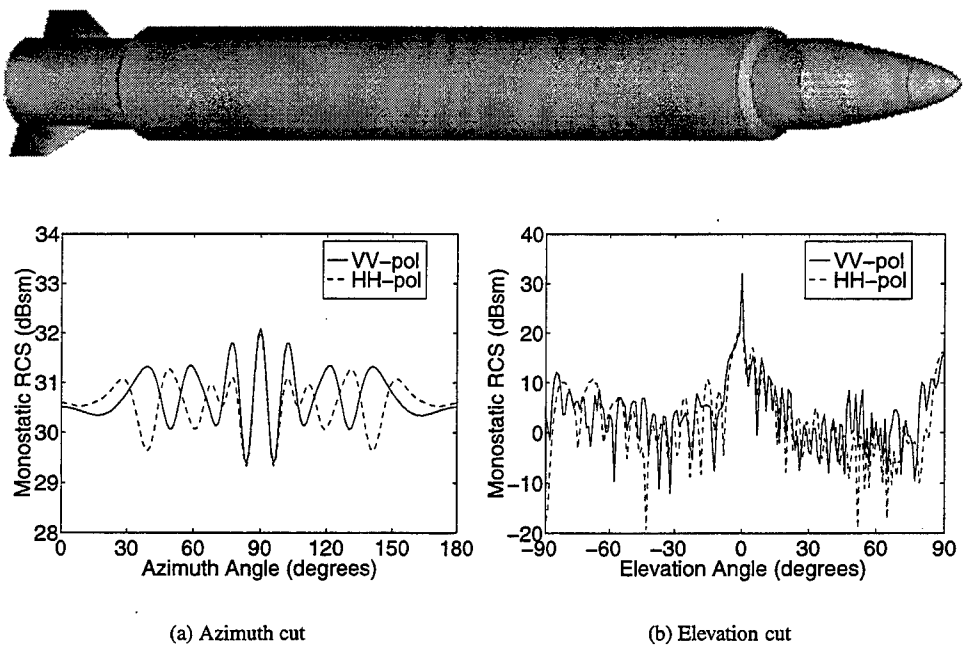


Figure 7: RCS of a missile with two fins at 1 GHz. The missile is 12.5 m ( $41.7\lambda$ ) long and has a radius of 0.625 m ( $2.1\lambda$ ). The fins are trapezoidal with a height of 0.375 m ( $1.25\lambda$ ), bases of 1 m ( $3.33\lambda$ ) and 0.5 m ( $1.67\lambda$ ), and a thickness of 0.01 m ( $0.03\lambda$ ); they are located at azimuth angles  $90^\circ$  and  $-90^\circ$ .

## 4 Summary

The hybrid method is a useful extension of the FEM/BOR capability described in [4]. While rigorous computation of scattering from a BOR with appendages requires a 3-D computational method, the hybrid method separates the BOR part of the problem from the appendages. The rotational symmetry of the BOR part of the problem can then be exploited for computational efficiency while only the appendage part, which is typically much smaller than the BOR part, requires a 3-D method. Numerical results show the impact of the appendages on the scattering from the entire structure and verify the validity and capability of the hybrid method.

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# IMPEDANCE OF A HALF-WAVE DIPOLE OVER A FINITE GROUND PLANE

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## ABSTRACT

A study has been carried out on the impedance of a half-wave dipole above a finite ground plane. The Method of Moment code, NEC-4, has been used and it has been found that for ground planes which are greater than 1.0 by 1.0 wavelengths, the changes from the effect with an infinite plane are negligible. With very small planes, the current distribution across the ground plane and the impedance are substantially different from those with larger ground planes.

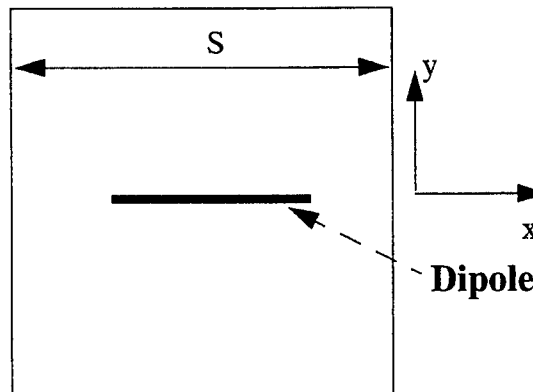
## 1. INTRODUCTION

There is a dearth of information in the open literature on the effect of a finite ground plane on the impedance of a nearby halfwave dipole although several authors have considered the effect of finite sheets on radiation patterns. We examine the general problem of impedance as a function of sheet size and dipole position relative to the edges of the sheet. Dipole impedances have been calculated using the Method of Moments Code, NEC-4 [1], for various dipole positions over square sheets of side 0.5, 1.0, 2.0 and  $2.5\lambda$  and are examined along with theoretical considerations.

## 2. COMPUTATIONS

The dipole was of diameter  $0.002\lambda$  (2mm) and was  $0.5\lambda$  (500mm) long. Spacing from the ground plane was  $0.221\lambda$  (221 mm). Throughout this paper, all dimensions are quoted in mm and in wavelengths at 300 MHz. The RF performance was examined over the frequency band 270-330 MHz.

Square sheets of size 0.5, 1.0, 2.0 and  $2.5\lambda$  (500 to 2500 mm) were tested. The dipole position, initially in the centre of the sheet, was altered in steps of  $0.25\lambda$  (250 mm) in either the x or y directions. The dipole lay along the x axis (Figure 1).



**Figure 1 Geometry of Dipole over a Finite Square Ground Plane. S was varied between  $0.5$  and  $2.5\lambda$ .**

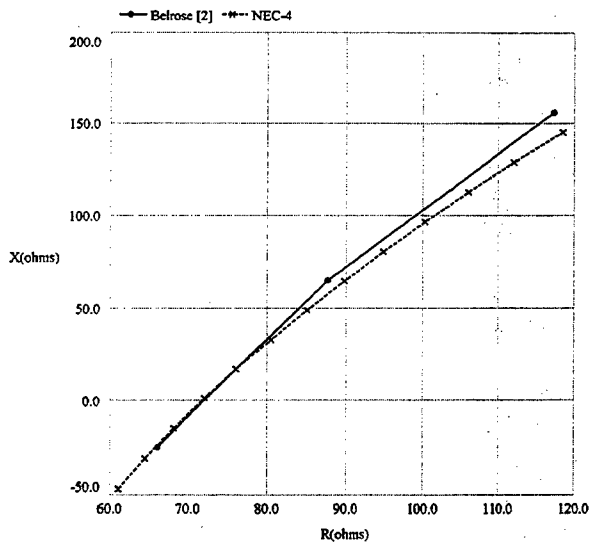
The ground planes were modelled as a square mesh with segments which were  $0.05\lambda$  long (50 mm) and had a radius of  $0.008\lambda$  (8.0 mm) giving a length to diameter ratio of 3.125 which is greater than the required lower limit of 2.0. This diameter is derived from the equal area rule but a convergence test was also applied (Section 7).

The dipole was modelled as 21 perfectly conducting linear segments in a length of 0.5 wavelengths (500 mm); each segment had a diameter of  $0.002\lambda$  (2 mm). The excitation was a voltage source of 1 volt applied to the centre segment. Trials were carried out with 51 segments for the dipole in free space but the results were very similar to those with 21 segments (Section 7).

Computations were carried out with both double and single precision versions of NEC-4 (Section 7).

## 3. FREE SPACE

An initial run was made with the dipole with no ground plane present. Table 1 gives the impedance values, the resonance frequency being just below 285 MHz where the length would have been  $0.473\lambda$ .



**Figure 2 Reactance against Resistance for a halfwave dipole in free space The NEC-4 values are indicated at 5.0 MHz steps between 270 and 330 MHz (Table 1).**

**Table 1 Impedance of dipole in free space (Dipole  $0.002\lambda$  diameter,  $0.5\lambda$  long at 300 MHz)**

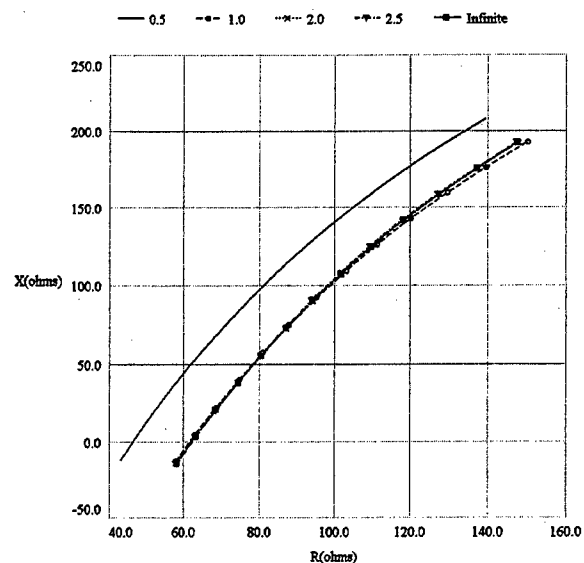
F MHz	NEC-4		Belrose [2]	
	R (ohms)	X (ohms)	R (ohms)	X (ohms)
270	61.1	-46.9	66.1	-25.0
275	64.5	-30.9		
280	68.2	-14.9		
285	72.1	1.02		
290	76.1	16.9		
295	80.5	32.8		
300	85.0	48.7	87.6	65.0
305	89.8	64.6		
310	94.9	80.5		
315	100.3	96.5		
320	106.0	112.6		
325	112.1	128.7		
330	118.5	145.0	117.4	155.8

Figure 2 compares the computed impedances and those calculated from Equations 15.16 and 15.17 of Belrose, [2] which are for a monopole. These values were doubled to obtain the numbers for a dipole shown in Figure 2 and Table 1. The agreement is good. It implies that the antenna is about 3% longer electrically than its dimensions would suggest from Belrose. In their Figure 13.31, showing calculated values of resistance at resonance as a function of wavelength/diameter, Schelkunoff and Friis, [3], give a value of 64 ohms for a dipole of the dimensions used here whereas NEC-4 and Belrose both give 72 ohms.

#### 4. GROUND PLANE

Figure 3 to Figure 5 show the impedance for the central and the two extreme positions. The values are indicated at 5.0 MHz steps between 270 and 330 MHz.

For sheets of side  $1.0\lambda$  and larger, there is no significant change in impedance with sheet size when the dipole is centrally placed or when it is over a parallel edge. When the dipole is moved longitudinally so that its centre is on the edge, there is more variation with sheet size as Figure 5 demonstrates. Results for the  $\lambda/2$  square sheet also show significantly lower resistance values. The impedance values for an infinite ground plane have been included in Figure 3 and show that, for plates of side greater than  $1.0\lambda$ , the results have converged. For the larger sheets, there does appear to be a very slight cyclical variation with dipole position. This is shown in Table 2 for 300 MHz on a  $2.5\lambda$  square sheet.



**Figure 3 Impedance of dipole (Central position) between 270 and 330 MHz**

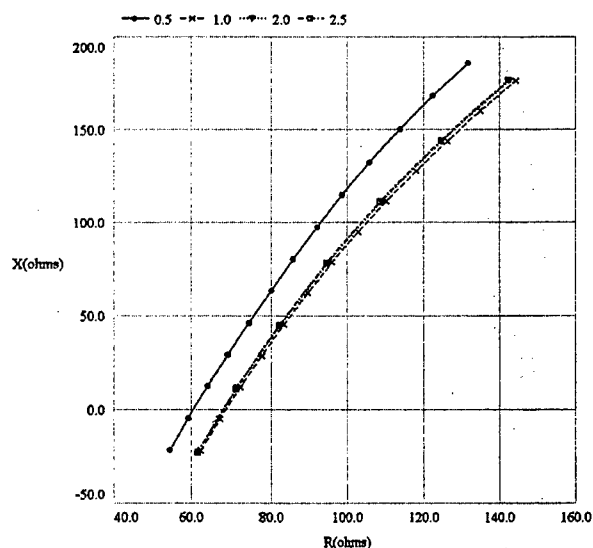


Figure 4 Impedance of dipole (over parallel edge) between 270 and 330 MHz

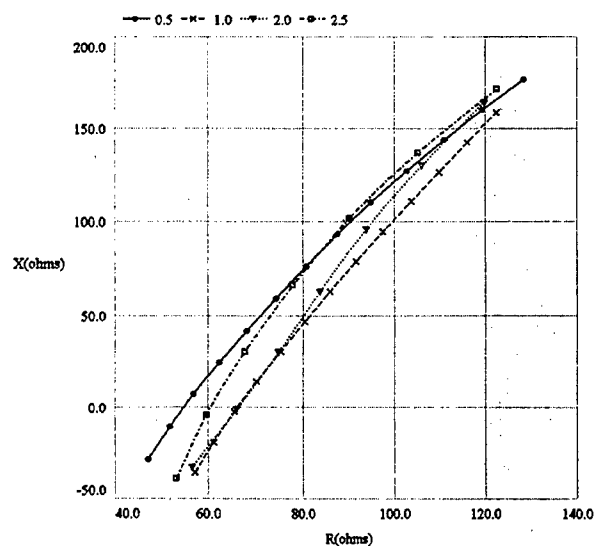


Figure 5 Impedance of dipole (over perpendicular edge) between 270 and 330 MHz.

Table 2 Impedance of dipole above ground plane of side  $2.5\lambda$  as a function of position off-centre. (Frequency 300 MHz)

$y/\lambda$	$x/\lambda = 0$	$x/\lambda$	$y/\lambda = 0$
0	$93.9 + j 90.2$	0	$93.9 + j 90.2$
0.25	$94.0 + j 90.2$	0.25	$94.1 + j 90.3$
0.5	$93.9 + j 90.1$	0.5	$93.8 + j 90.0$
0.75	$94.1 + j 90.1$	0.75	$94.7 + j 90.5$
1.0	$92.6 + j 90.6$	1.0	$87.7 + j 91.6$
1.25	$94.5 + j 77.8$	1.25	$77.8 + j 66.0$

## 5. CURRENT FLOW

As noted above, the values of the impedance for the smallest ground plane ( $0.5$  by  $0.5 \lambda$ ) were quite different from those of the larger ground plane.

The currents on the ground planes at intervals of  $0.25$  wavelengths have been plotted for a dipole centrally placed. The abscissa (current magnitude) has been multiplied by  $10^4$  for convenience.

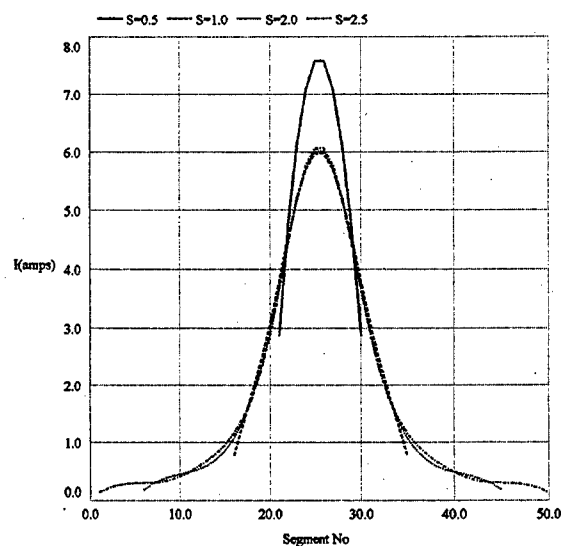


Figure 6 Current Magnitude ( $\times 10^4$ ) on a Ground Plane parallel to the dipole and immediately below the dipole

Figure 6 shows the currents on the ground plane immediately below the dipole for several ground plane sizes while Figure 7 and Figure 8 show the currents  $0.25$  and

0.5 wavelength away from the centreline underneath the dipole. The most notable feature is that the current distribution is remarkably alike for ground planes with sides 1.0 to 2.5 wavelengths while the ground plane of side 0.5 wavelength is very different.

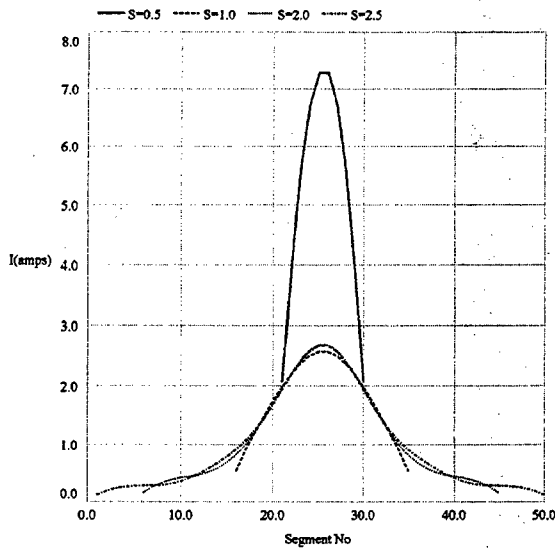


Figure 7 Current Magnitude ( $\times 10^4$ ) on a Ground Plane parallel to the dipole and offset by 0.25 wavelength from the dipole

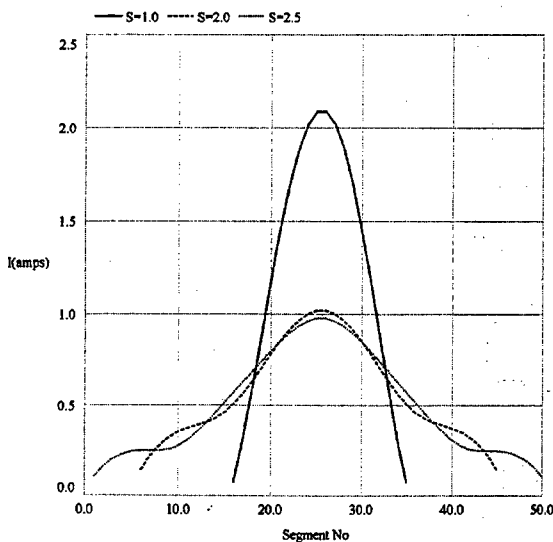


Figure 8 Current Magnitude ( $\times 10^4$ ) on a Ground Plane parallel to the dipole and offset by 0.5 wavelengths from the dipole

## 6. CHECKS

It was possible to provide some corroboration for the predictions by computing the mutual impedance provided when a half-wave dipole is over an infinite ground plane. The mutual impedance is then the impedance of the dipole alone minus the mutual impedance of the dipole over an infinite ground plane. These are compared with results from Smith [5] and STC Data [4] in Table 3. These results are in better agreement with the STC data.

Since the image of the dipole in the ground plane is negative, the computed figures are in opposite senses to those from two parallel dipoles at the same spacing as the dipole and its image. The computed figures have been reversed in sign for comparison with other sets of data.

Table 3 Mutual Impedance of a Dipole above an Infinite Ground Plane

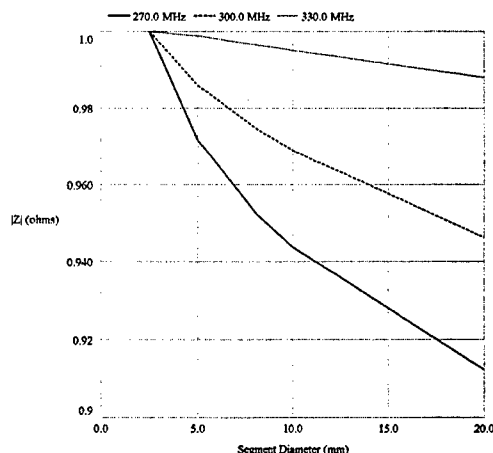
$\Delta F$ MHz	NEC-4		STC data		R A Smith	
	R	jX	R	jX	R	jX
-5.0	1.1	-31.4	1.1	-34.8	4.0	-36.9
-1.0	-1.5	-33.6	-1.5	-36.4	1.5	-36.3
+2.0	-3.7	-35.2	-3.2	-35.5	-0.3	-36.0
+4.0	-5.3	-36.2	-4.2	-35.1	-1.3	-35.7
+6.0	-7.0	-37.2	-5.2	-34.5	-2.5	-35.3
+7.0	-7.9	-37.8	-6.0	-34.1	-3.1	-35.1
RMS Error in NEC			1.2	2.6	3.8	3.0

## 7. CONVERGENCE TESTS

Several convergence tests were carried out on the input data.

- 1) Single and double precision versions of NEC-4 were run on all the examples for a ground plane with a side of  $0.5 \lambda$ . The changes in all values of impedance were less than 0.001%.
- 2) The radius of segments in the ground plane was varied between 20 and 2.5 mm with the segment length held constant at 50 mm (Figure 9). The value chosen, 8.0 mm, was based on the equal area rule and computed results agreed with other results to within 1 or 2 ohms.
- 3) The number of segments in the dipole was varied from 21 to 51. Differences of less than 1.0 degree

in phase and 1% in impedance magnitude were observed.



**Figure 9** Impedance for three frequencies for dipole in centre of  $0.5\lambda$  ground plane with the segment radii of the ground plane varied. The impedance magnitude is shown normalised to the value at a segment radius of 2.5 mm

## 8. CONCLUSIONS

- 1) The impedance of an isolated  $0.5\lambda$  dipole obtained by NEC is closely modelled by the Belrose [2] formula for a dipole of the same dimensions. The difference of 3% in frequency may arise from small errors in Belrose's determination of electrical length which was empirical. Alternatively it may represent a genuine difference between the two models. It has been noted (Section 7) that small changes in the NEC-4 model resulted in changes in the impedance of up to 1% in magnitude.
- 2) When mounted above the centre of a finite ground plane, the dipole impedance does not alter significantly for sheets of  $1\lambda$  side or larger. The resistance of a dipole above a sheet of side  $0.5\lambda$  is significantly lower. The plots of current flow in Figure 6 to Figure 8 support the impedance results.
- 3) When the dipole is over the parallel edge of the sheet, the general picture is similar to that of the central position.
- 4) When the dipole centre is over the perpendicular

edge, there is more variation with sheet size.

- 5) The impedance on larger sheets appears to be more sensitive to movement along the dipole axis than to movement in the perpendicular direction.

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## The Design of Planar Slot Arrays Revisited

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### Abstract

Design procedures for planar waveguide slot arrays have been known and used for years, but the inexperienced designer is often faced with a number of practical problems when trying to implement them, especially in the case of larger arrays consisting of sub-arrays. Some helpful hints concerning the numerical implementation are provided. These include an effective procedure to avoid the necessity of good initial guesses for the unknown dimensions and recommendations for the subdivision of the nonlinear equations into smaller groups. Other practical aspects which are not explicitly defined elsewhere in the literature, are also addressed.

### 1. INTRODUCTION

Planar slot arrays are key components of high performance airborne radar systems, and also find application in modern microwave communication systems. A planar array consists of a number of slotted waveguides arranged side-by-side, while the most popular way of feeding the individual branches is through the use of crossed waveguide couplers with centred-inclined slots. For large arrays, the concept of sub-arraying is often used to improve the usable frequency bandwidth, and also to reduce the sensitivity of antenna performance on dimensional tolerances. Each sub-array is then fed by a main line, while the individual main lines are fed by a beam forming network. Slotted array seeker antennas also make use of sub-arraying, where a monopulse comparator network effectively feeds the four quadrants of the antenna either in-phase or out of phase with respect to each other.

Design procedures for the synthesis of linear slot arrays were proposed by Elliott [1-3], and were later extended to cover planar arrays [4]. Other contributions to the field of array design include those of [5] and [6]. Even though these techniques have been known and used for years, the inexperienced designer is still faced with a number of practical problems when trying to implement them, especially in the case of larger arrays consisting of sub-arrays. In this paper, some helpful hints concerning the numerical implementation are provided, and other practical aspects which are not explicitly defined elsewhere in the literature are addressed.

The design procedure involves the repeated solution of sets of nonlinear equations in order to iteratively compute the offsets and lengths of the radiating slots, as well as the inclination angles and lengths of coupling slots. The successful numerical solution of sets of nonlinear equations is usually dependent on the quality of the initial guesses for the unknowns, the number

of unknowns and also on the nature of the nonlinear equations. In practice, specifying suitable starting values may require considerable effort. In this paper, a simple but highly effective method, which is less dependent on good initial guesses and a priori knowledge about the eventual geometry, is proposed.

It is known that the solution obtained for a specific design is not unique, due to a number of degrees of freedom. This aspect is usually addressed by fixing some of the dimensions. A different way of accommodating the redundancy is proposed. It naturally divides the nonlinear equations into smaller groups by effectively decoupling them into semi-independent sets which are solved individually during each iteration.

There also seems to be some uncertainty regarding the correct orientation of coupling slots, especially when the array has more than one feed line. Unequivocal relations for the signs of radiating slot offsets and coupling slot angles are thus provided. Finally, some guidelines on the specifications for beam-forming networks required by multi-sub-array systems are given.

### 2. THEORETICAL BACKGROUND

Consider a planar array consisting of a total of  $S$  sub-arrays. The  $s$ th sub-array shown in Fig. 1 consists of a total of  $T_s$  branch lines, while the  $t$ th branch line has a total of  $N_{s,t}$  slots. The slots are resonantly spaced, i.e.  $d = \lambda_g/2$  and  $d_0 = \lambda_g/4$  at the design frequency,  $f_0$ . The inter-slot spacing should be larger than the waveguide width,  $a$ , and for a waveguide filled with a dielectric material of relative permittivity  $\epsilon_r$ , the value of  $a$  should be between the limits  $\lambda_0/2\sqrt{\epsilon_r} \leq a \leq \lambda_0/\sqrt{2\epsilon_r}$ , where  $\lambda_0$  is the free space wavelength. The thickness of the septum separating neighboring branches is thus  $t_0 = \lambda_g/2 - a$ . The waveguide height,  $b$ , may be chosen arbitrarily. The sub-array is fed by means of a main line, which is connected to the different branch lines via centred-inclined coupling slots. The coupling slot feeding the  $t$ th branch line of the  $s$ th sub-array has an inclination angle of  $\theta_{s,t}$  and a slot length of  $l_{s,t}$ . The angle  $\theta_{s,t}$  is taken as positive in a clockwise direction. This coupling slot is located between the  $k_{s,t}$ th and the  $(k_{s,t}+1)$ th slot of the  $t$ th branch line. The term  $\hat{u}_s = u_s \hat{x}$  is the unit vector in the direction toward the shorted end of the main line, so that  $u_s = \pm 1$  for sub-arrays fed from the bottom and the top, respectively. The complete array has a total number of

$$M = \sum_{s=1}^S \sum_{t=1}^{T_s} N_{s,t} \text{ slots.}$$



Two distinct radiating slot indexing conventions are used:

- (a) A local numbering system with a triple index  $(s, t, n)$  denoting the  $n$ th slot in the  $t$ th branch line of the  $s$ th sub-array. The slot has a width of  $w$ , an offset of  $x_{s,t,n}^{\text{off}}$  relative to the center line of the branch line, and a slot length of  $L_{s,t,n}$ . Slots in a branch line are numbered from left to right, irrespective of whether the sub-array is fed from the bottom (as in Fig. 1) or from the top. Branch lines are numbered such that the line closest to the feeding end is denoted by  $t = 1$  and the line closest to the shorted end of the main line by  $t = T_s$ .
- (b) A global numbering system with a single index,  $i$ . The slot with local index  $(s, t, n)$  has a global index of

$$i = \sum_{u=1}^{s-1} \sum_{t=1}^{T_u} N_{u,t} + \sum_{v=1}^{t-1} N_{s,v} + n.$$

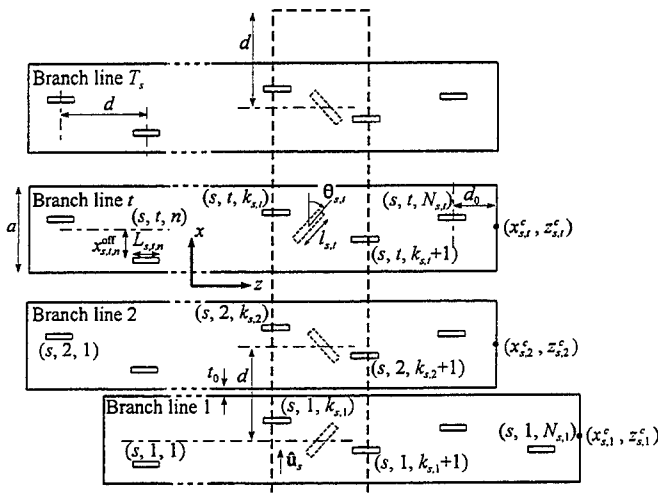


Fig. 1 Geometry of the  $s$ th sub-array.

The equivalent network for the  $t$ th branch line of the  $s$ th sub-array is shown in Fig. 2, where  $y_{s,t,n}^a$  is the normalized active admittance of the  $n$ th slot. The design procedure involves the repeated solution of a set of nonlinear equations in order to determine the slot offsets and lengths. The nonlinear equations are related to the so-called design equations, given by [2]

$$y_{s,t,n}^a = K_1 f_{s,t,n} \frac{V_{s,t,n}^{\text{slot}}}{V_{s,t,n}} \quad (1)$$

and [3]

$$\frac{1}{y_{s,t,n}^a} = \frac{1}{y_{s,t,n}^{\text{self}}} + \frac{1}{(f_{s,t,n})^2 K_2} \sum_{j=1, j \neq i}^M \frac{V_j^{\text{slot}}}{V_i^{\text{slot}}} g_{ji} + \frac{1}{(f_{s,t,n})^2 K_3} \left[ \frac{V_{s,t,n-1}^{\text{slot}}}{V_{s,t,n}^{\text{slot}}} h_{s,t,n} h_{s,t,n-1} + \frac{V_{s,t,n+1}^{\text{slot}}}{V_{s,t,n}^{\text{slot}}} h_{s,t,n} h_{s,t,n+1} \right] \quad (2)$$

For the slot with local index  $(s, t, n)$  and global index  $i$ , the slot voltage is denoted as  $V_{s,t,n}^{\text{slot}}$  or  $V_i^{\text{slot}}$ , while the corresponding

voltage in the equivalent network is  $V_{s,t,n}$ . The term  $y_{s,t,n}^{\text{self}} = y^{\text{self}}(x_{s,t,n}^{\text{off}}, L_{s,t,n})$  represents the normalized self-impedance of this slot. The function  $g_{ji}$  is given by [2], which is dependent on the coordinates of the centers of the  $i$ th and the  $j$ th slots. Let  $(x_i, z_i)$  the coordinates for the  $i$ th slot in the global coordinate system. In terms of the local coordinates, it is given by

$$(x_i, z_i) = (x_{s,t}^c + x_{s,t,n}^{\text{off}}, z_{s,t}^c - d_0 - N_{s,t}d + nd) \quad (3)$$

where  $(x_{s,t}^c, z_{s,t}^c)$  are the coordinates of the center point of the right edge of the  $t$ th branch line. The functions  $f_{s,t,n}$  and  $h_{s,t,n}$  may be obtained from [2] and [3] by making the substitutions  $2l_n = L_{s,t,n}$  and  $x_n = x_{s,t,n}^{\text{off}}$ . For  $n = 1$  or  $n = N_{s,t}$ , the undefined terms in (2) such as  $V_{s,t,0}^{\text{slot}}$ ,  $V_{s,t,N_{s,t}+1}^{\text{slot}}$ ,  $h_{s,t,0}$  and  $h_{s,t,N_{s,t}+1}$  may be found through the application of image theory. The constants  $K_1$ ,  $K_2$  and  $K_3$  may also be obtained from [2] and [3].

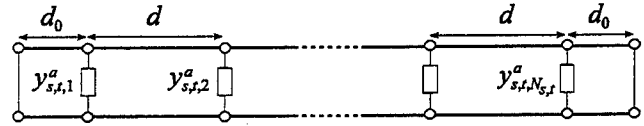


Fig. 2 Equivalent transmission line network for the  $t$ th branch line of the  $s$ th sub-array.

Let the voltages across the shunt elements in Fig. 2 be given by

$$V_{s,t,n} = q_{s,t} V_{s,t}^0 (-1)^{k_{s,t}-n} \quad (4)$$

The term  $q_{s,t}$  is an arbitrary constant equal to  $\pm 1$ , depending on whether the offset of the slot with index  $(s, t, k_{s,t})$  is to be positive or negative. These voltages are arbitrarily chosen to be real valued. Then  $V_{s,t}^0$  is a positive real number representing the magnitude of the voltages on the equivalent circuit of the  $t$ th branch line of sub-array  $s$ .

The series impedances of the equivalent circuit for the main line in Fig. 3 are given by [4]

$$z^{(s,t)} = (\kappa^{(s,t)})^2 y_{\text{in}}^{(s,t)} \quad (5)$$

where  $y_{\text{in}}^{(s,t)}$  is the normalized input admittance of the  $t$ th branch line of the  $s$ th sub-array, given by

$$y_{\text{in}}^{(s,t)} = \sum_{n=1}^{N_{s,t}} y_{s,t,n}^a \quad (6)$$

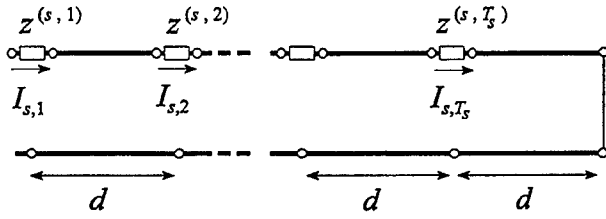


Fig. 3 Equivalent network for the main line of the  $s$ th subarray.

The term  $\kappa^{(s,t)}$  is the coupling coefficient defined by

$$\kappa^{(s,t)} = \sqrt{\frac{S_{11}^{(s,t)}}{1 - S_{11}^{(s,t)}}} \quad (7)$$

while  $S_{11}^{(s,t)} = S_{11}(\theta = |\theta_{s,t}|, l = l_{s,t})$  is the scattering matrix component of the crossed waveguide couplers formed at the junction of the  $s$ th mainline and the  $t$ th branch line, as shown in Fig. 4. The normalized input impedance of the  $s$ th main line is then given by

$$z_{in}^{(s)} = \sum_{t=1}^{T_s} z^{(s,t)} = \sum_{t=1}^{T_s} (\kappa^{(s,t)})^2 y_{in}^{(s,t)} \quad (8)$$

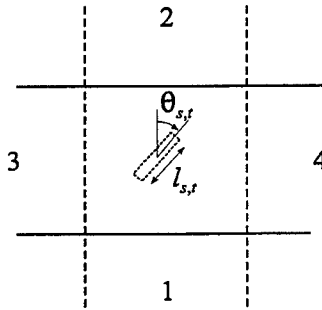


Fig. 4 Crossed-guide coupler geometry for the main line of the  $s$ th subarray and the  $t$ th stick.

Due to the fact that the coupling slots are also resonantly spaced (i.e.  $d = \lambda_g/2$  at the design frequency,  $f_0$ ), the current  $I_{s,t}$  in Fig. 3 is given by

$$\begin{aligned} I_{s,t} &= (-1)^{t-1} I_{s,1} \\ &= (-1)^{t-1} \frac{2a^{(s)}}{\sqrt{Z_0}(z_{in}^{(s)} + 1)} \end{aligned} \quad (9)$$

where  $I_{s,1}$  is the current at the first series element, while  $a^{(s)}$  is the incident wave intensity. Suppose the input impedance of each sub-array is purely resistive. Then  $z_{in}^{(s)} = r_{in}^{(s)}$ , where  $r_{in}^{(s)}$  is the chosen normalized input resistance of the  $s$ th sub-array as seen from the first series impedance in the main line equivalent circuit. The relation between the current at the

series impedance representing the  $t$ th coupling slot in the main line,  $I_{s,t}$ , and the voltage across the  $k_{s,t}$ th element in the equivalent network for the  $t$ th branch line,  $V_{s,t,k_{s,t}}$ , is given by

$$V_{s,t,k_{s,t}} = u_s \frac{\theta_{s,t}}{|\theta_{s,t}|} \kappa^{(s,t)} I_{s,t} \quad (10)$$

One design objective is to have all the slot voltages in phase. From equation (1), it follows that the argument of the term  $V_{s,t,n}/f_{s,t,n}$  should be uniform. In agreement with the choice of real-valued equivalent network voltages, this phase reference is taken to be zero. In a specific branch line, this is accomplished by alternating the offsets of the slots on opposite sides of the center line. The sign of the term  $f_{s,t,k_{s,t}}$  will be equal to the value of  $q_{s,t}$ , which implies that the sign of  $V_{s,t,k_{s,t}}$  should also be identical to the value of  $q_{s,t}$ . This agrees with the choice in (4). For resonant coupling slots,  $\kappa^{(s,t)}$  is purely real, while (9) and (10) dictate that  $a^{(s)}$  should also be real valued. Consequently, the orientations of the coupling slots are given by

$$\frac{\theta_{s,t}}{|\theta_{s,t}|} = (-1)^{t-1} u_s q_{s,t} \frac{a^{(s)}}{|a^{(s)}|} \quad (11)$$

Substituting (9) and (11) into (10) gives

$$V_{s,t,k_{s,t}} = q_{s,t} \kappa^{(s,t)} \frac{2|a^{(s)}|}{\sqrt{Z_0}(r_{in}^{(s)} + 1)} \quad (12)$$

A comparison of equations (4), (9) and (12) then yields the simple relation of [4], given by

$$V_{s,t}^0 = \kappa^{(s,t)} I_s^0 \quad (13)$$

where  $I_s^0 = |I_{s,t}|$ .

### 3. DERIVATION OF THE SETS OF NONLINEAR EQUATIONS

Consider the  $s$ th sub-array. Select the  $k_{s,t}$ th slot branch line  $t$  to serve as the reference slot. From equations (1) and (4), it is found that

$$\frac{y_{s,t,n}^a}{y_{s,t,k_{s,t}}^a} = \frac{f_{s,t,n} V_{s,t,n}^{\text{slot}} (-1)^{n-k_{s,t}}}{f_{s,t,k_{s,t}} V_{s,t,k_{s,t}}^{\text{slot}}} \quad (14)$$

Note that the ratio in (14) is only dependant on the slot offsets and lengths in branch line  $t$  of sub-array  $s$ . Equation (14) does not provide a sufficient number of equations to uniquely solve for the unknown slot offsets and lengths in the branch. It has been suggested that the slot which is to have the greatest slot offset be singled out and assigned an offset near the upper limit of reliable input data [4]. However, it is often difficult to identify this slot without some degree of a priori knowledge of the eventual geometry, especially in cases of uniform

excitation. A different approach is to specify the input admittance of each branch line to be purely real and with a normalized input admittance of  $g_{in}^{(s,t)}$ . Equations (6) and (14) may then be used to construct the following set of  $2N_{s,t}$  nonlinear equations for the unknown slot offsets and lengths of branch line  $t$  in sub-array  $s$ :

$$\text{Re} \left[ \frac{y_{s,t,n}^a}{y_{s,t,k_{s,t}}^a} - \frac{f_{s,t,n} V_{s,t,n}^{\text{slot}} (-1)^{n-k_{s,t}}}{f_{s,t,k_{s,t}} V_{s,t,k_{s,t}}^{\text{slot}}} \right] = 0$$

$$\text{Im} [y_{s,t,n}^a] = 0 \quad n = 1, 2, \dots, N_{s,t} \quad n \neq k_{s,t} \quad (15)$$

$$g_{in}^{(s,t)} - \text{Re} \left[ \sum_{n=1}^{N_{s,t}} y_{s,t,n}^a \right] = 0$$

where  $y_{s,t,n}^a$  is calculated using equation (2).

Comparing the active admittances of the  $k_{s,t}$ th slot of the  $t$ th branch line and the  $k_{s,1}$ th slot of the first branch line, using equations (1) and (12), results in

$$\frac{y_{s,t,k_{s,t}}^a}{y_{s,1,k_{s,1}}^a} = \frac{|f_{s,t,k_{s,t}}| V_{s,t,k_{s,t}}^{\text{slot}} \kappa^{(s,1)}}{|f_{s,1,k_{s,1}}| V_{s,1,k_{s,1}}^{\text{slot}} \kappa^{(s,t)}} \quad (16)$$

If the slot offsets and lengths of the reference slots are known, equations (8) and (16) may be used to construct the following set of  $2T_s$  nonlinear equations for the magnitude of the inclination angles and coupling slot lengths of main line  $s$ :

$$\text{Re} \left[ \frac{y_{s,t,k_{s,t}}^a}{y_{s,1,k_{s,1}}^a} - \frac{|f_{s,t,k_{s,t}}| V_{s,t,k_{s,t}}^{\text{slot}} \kappa^{(s,1)}}{|f_{s,1,k_{s,1}}| V_{s,1,k_{s,1}}^{\text{slot}} \kappa^{(s,t)}} \right] = 0 \quad t = 2, 3, \dots, T_s$$

$$\text{Im} [\kappa^{(s,t)}] = 0 \quad t = 1, 2, \dots, T_s \quad (17)$$

$$\text{Re} \left[ r_{in}^{(s)} - \sum_{t=1}^{T_s} (\kappa^{(s,t)})^2 g_{in}^{(s,t)} \right] = 0$$

where  $\kappa^{(s,t)}$  is calculated using (7).

#### 4. EFFECTIVE IMPLEMENTATION OF THE STARTING VALUE PROBLEM

Depending on the nature of a set of nonlinear equations, their numerical solution may be highly dependent on the quality of the initial guesses for the unknowns, as well as the number of unknowns. In the previous section, the equations were subdivided into smaller sets, but the designer is still faced with finding good starting values of the unknown slot dimensions.

For larger arrays, the evaluation of the external mutual coupling terms  $g_{ji}$  is generally the most time consuming step in the numerical implementation of a design procedure. For

practical purposes, an iterative approach for the design of linear arrays was proposed in [2], where during each iteration, these terms are computed using the current set of slot dimensions and are assumed to remain constant while calculating new dimensions by way of solving of a set of nonlinear equations. It was also suggested in [4] that the external and internal mutual coupling terms be set to zero for the first iteration. Due to the simplified nature of the resulting nonlinear equations, this negates the need for good initial guesses for the unknown as required by nonlinear equation solvers. Arbitrary starting values within the realistic range of slot dimensions may then be specified, while the nonlinear equation solver routine is usually successful in finding a solution. During the second iteration, the computed slot dimensions for the zero mutual coupling case are used to re-evaluate the mutual coupling terms, and the same dimensions are employed as starting values for the subsequent solution of the nonlinear equations. This approach is entirely effective for linear arrays. This is due to the fact that the mutual coupling between slots that are approximately axially aligned is relatively small, and therefore the abrupt inclusion of the mutual coupling contributions during the second iteration only has a secondary effect, and the changes in slot dimensions never become excessive.

For planar arrays the displacement between certain slots is also lateral, which causes the external mutual coupling to have a more pronounced effect on the slot dimensions. The examples in [3] bare evidence of this. Even though a uniform excitation for all slots was specified, the mutual coupling causes large variations in slot offsets, while in the absence of mutual coupling, all slots have the same lengths and offset magnitudes. Implementation of the approach proposed for linear arrays may result in numerical instability. Problems generally do not occur when solving for the zero mutual coupling case during the first iteration. However, during the second iteration where the zero mutual coupling dimensions (which may differ appreciably from the final results) are used to first calculate  $g_{ji}$  and then also as starting values for the subsequent solving of the nonlinear equations, the procedure often fails.

It is therefore proposed that the procedure be modified by retaining the first iteration with zero mutual coupling, but instead of abruptly introducing the effects of mutual coupling during the second iteration, that its effects be included gradually. This is achieved by weighing the values of  $g_{ji}$  and  $h_{s,t,n}$  with a weighting factor increasing from 0 to 1 during the first half of a chosen total number of iterations. Consequently, the first iteration will be performed with zero mutual coupling, while the full effect of the mutual coupling is only included during the second half of the iterations. This approach generously compensates for a possible increase in the total number of iterations by being very robust and reliable. The ease of implementation of this procedure is also an advantage

over other approaches which merely rely on good starting values that are obtained by for instance first considering an infinite array [6].

## 5. BEAM-FORMING NETWORK

An array consisting of a number of sub-arrays would naturally require a power splitter or beam-forming network to distribute the power from the antenna input to the multiple sub-array main lines. A complete design should therefore also include the specifications of the power splitter network. A comparison of the active admittances of the  $k_{s,1}$ th slot of the first branch line of sub-array  $s$  and of the  $k_{1,1}$ th slot of the first branch line of sub-array 1 using equations (1) and (12) yields the following relative amplitudes of the incident wave intensities for the sub-arrays:

$$\frac{|a^{(s)}|}{|a^{(1)}|} = \frac{|f_{s,1,k_{s,1}}| V_{s,1,k_{s,1}}^{\text{slot}} \kappa^{(1,1)} y_{1,1,k_{1,1}}^a (r_{\text{in}}^{(s)} + 1)}{|f_{1,1,k_{1,1}}| V_{1,1,k_{1,1}}^{\text{slot}} \kappa^{(s,1)} y_{s,1,k_{s,1}}^a (r_{\text{in}}^{(1)} + 1)} \quad (18)$$

The phase difference between  $a^{(s)}$  and  $a^{(1)}$  is either  $0^\circ$  or  $180^\circ$ . The phase relation is chosen in accordance with the phase properties of the beam-forming network to be used. Together with (18), the ratio  $a^{(s)}/a^{(1)}$  is thus completely specified. In general, an array consisting of  $S$  sub-arrays would require a splitter with  $P = S + 1$  ports. For the sake of clarity, it is assumed that each of the first  $S$  ports is connected to its corresponding sub-array, while the  $P$ th port serves as the common feed port. From the definition of the scattering parameters, it follows that

$$[S^{\text{split}}][a^{\text{split}}] = [b^{\text{split}}] \quad (19)$$

$[S^{\text{split}}]$  is the  $P \times P$  scattering parameter matrix of the network, specified with the phase reference of port 1 to port  $S$  at the center of the first coupling slot of the main line it is connected to. Since the first  $S$  ports of the splitter network are terminated in the input impedances of the sub-arrays, it follows that

$$a_s^{\text{split}} = \Gamma_{\text{in}}^{(s)} b_s^{\text{split}} \quad s = 1, 2, \dots, S \quad (20)$$

where

$$\Gamma_{\text{in}}^{(s)} = \frac{r_{\text{in}}^{(s)} - 1}{r_{\text{in}}^{(s)} + 1} \quad (21)$$

The wave emanating from the  $s$ th port is fed directly into the main line of the  $s$ th sub-array, while the antenna should normally be matched at port  $P$ . This implies that

$$b_s^{\text{split}} = \begin{cases} a^{(s)} & s = 1, 2, \dots, S \\ 0 & s = P \end{cases} \quad (22)$$

For a lossless splitter network, the conservation of power condition requires that

$$\sum_{s=1}^P (|a_s^{\text{split}}|^2 - |b_s^{\text{split}}|^2) = 0 \quad (23)$$

Using (22) and (23) together with the specification for  $a^{(s)}/a^{(1)}$  then gives

$$\frac{b_s^{\text{split}}}{a_p^{\text{split}}} = \begin{cases} \frac{\frac{a^{(s)}}{a^{(1)}} e^{j\zeta}}{\sqrt{\sum_{m=1}^S \left(\frac{a^{(m)}}{a^{(1)}}\right)^2 [1 - \Gamma_{\text{in}}^{(m)2}]}} & s = 1, 2, \dots, S \\ 0 & s = P \end{cases} \quad (24)$$

$\zeta$  is an arbitrary common phase term. The relations in (24) specify the requirements for the splitter network. The simplest way of realizing this would be to design the sub-arrays to be matched (i.e.  $r_{\text{in}}^{(s)} = 1$  and  $\Gamma_{\text{in}}^{(s)} = 0$ ), which would then require that

$$S_{sP}^{\text{split}} = \begin{cases} \frac{(a^{(s)}/a^{(1)}) e^{j\zeta}}{\sqrt{\sum_{m=1}^S (a^{(m)}/a^{(1)})^2}} & s = 1, 2, \dots, S \\ 0 & s = P \end{cases} \quad (25)$$

## 6. DESIGN PROCEDURE

Array design requires data on the self admittance of isolated radiating slots as a function of the slot offset and length, as well as scattering parameters for coupling slots in terms of the inclination angle and slot length. These may either be obtained through measurement or calculation. In the case of the latter, the formulations in [7] and [8] may be used to pre compute the databases, while a bivariate spline interpolation scheme as in [9] has proved to be very effective for the calculation of the terms  $y_{s,t,n}^{\text{self}} = y^{\text{self}}(x_{s,t,n}^{\text{off}}, L_{s,t,n})$  and  $\kappa^{(s,t)} = \kappa(|\theta_{s,t}|, l_{s,t})$ . The design procedure then consists of the following:

1. Specify the slot voltages  $V_{s,t,n}^{\text{slot}}$  for a desired radiation pattern.
2. Specify  $u_s$  for each sub-array, as well as  $q_{s,t}$  and  $k_{s,t}$  for each branch line.
3. Select realistic values for the input conductance of each branch line,  $g_{\text{in}}^{(s,t)}$ . A guideline is to choose a typical slot offset  $x_{\text{typ}}^{\text{off}}$  and to identify the corresponding resonant length for this offset,  $L_{\text{typ}}$ . A typical value for the input conductance would then be  $g_{\text{in}}^{(s,t)} = N_{s,t} \cdot y^{\text{self}}(x_{\text{typ}}^{\text{off}}, L_{\text{typ}})$ .
4. Specify the input resistance of each main line,  $r_{\text{in}}^{(s)}$ . Unless the circumstances dictate otherwise, the most convenient choice would be  $r_{\text{in}}^{(s)} = 1$ .
5. Select an appropriate number of iterations to be performed,  $C$ , where  $C \geq 10$  is an even-valued integer.
6. Select initial guesses for the lengths and offsets for the

radiating slots. An effective choice is to select  $x_{s,t,n}^{\text{off}} = (-1)^{k_{s,t,n}} q_{s,t} x_{\text{typ}}^{\text{off}}$  and  $L_{s,t,n} = L_{\text{typ}}$ .

7. Select initial guesses for the magnitude of the inclination angles and slot lengths for the coupling slots.
8. Set the iteration counter  $c = 1$ .
9. Calculate the global slot coordinates using equation (3).
10. Calculate the mutual coupling terms  $g_{ji}$  and  $h_{s,t,n}$ .
11. For the first half of the total number of iterations ( $c \leq C/2$ ), weigh the values of  $g_{ji}$  and  $h_{s,t,n}$  with a factor of  $2(c-1)/C$ , i.e. scale all the elements of  $g_{ji}$  to  $2(c-1)g_{ji}/C$  and the elements of  $h_{s,t,n}$  to  $2(c-1)h_{s,t,n}/C$ .
12. Set  $s = 1$ .
13. Set  $t = 1$ .
14. Solve the set of  $2N_{s,t}$  nonlinear equations in (15) for the unknown slot offsets and lengths of branch line  $t$  in sub-array  $s$ :  $(x_{s,t,n}^{\text{off}}, L_{s,t,n})$  where  $n = 1, 2, \dots, N_{s,t}$ .
15. Increment  $t$  and repeat step 14 until  $t = T_s$ .
16. Solve the set of  $2T_s$  nonlinear equations in (17) for the magnitude of the inclination angles and coupling slot lengths in main line  $s$ :  $(|\theta_{s,t}|, l_{s,t})$  where  $t = 1, 2, \dots, T_s$ .
17. Increment  $s$  and repeat steps 13-16 until  $s = S$ .
18. Increment  $c$  and repeat steps 9-17 until  $c = C$ .
19. Calculate the ratios of the wave intensities that a power splitter network needs to supply to the different sub-arrays from (18).
20. Specify the phase relation at each of the first  $S$  ports of the power splitter, i.e. whether sub-arrays are to be fed in phase or  $180^\circ$  out of phase relative to the excitation of the first sub-array, and use (11) to determine the orientation of the coupling slots.

## 7. DESIGN EXAMPLE

As an example, a small array similar to the one shown in Fig. 5 was designed. It consists of four sub-arrays, each having three branches and three slots per branch. A design frequency of  $f_0 = 9$  GHz was chosen, and half-height X-band waveguide of width  $a = 22.86$  mm and height  $b = 5.08$  mm was used for the branch guides and feed lines. A database for the properties of isolated radiating slots and inclined coupling slots was computed, using a slot width of  $w = 1.5875$  mm and a waveguide wall thickness of  $t = 1.27$  mm. A uniform excitation for all radiators was adopted. The array was designed to have a normalized conductance of  $g_{\text{in}}^{(s,t)} = 1.5$  for all branches, and a normalized input resistance of  $r_{\text{in}}^{(s)} = 1$  for each of the four sub-arrays. From the geometry, it follows that  $u_1 = u_2 = -1$ ,  $u_3 = u_4 = 1$ ,  $k_{1,t} = k_{4,t} = 1$ ,  $k_{2,t} = k_{3,t} = 2$  and  $q_{s,t} = -1$ . A total number of ten iterative steps were used during the design. The computed slot offsets and slot lengths for the radiating slots are shown in Table I, while the inclination angles and lengths of the coupling slots are listed in Table II. Due to the symmetry, sub-arrays 1 and 3 and sub-arrays 2 and 4 are mirror images of each other, and therefore corresponding slots will have the same magnitude of offset or

inclination angle, but with a difference in sign.

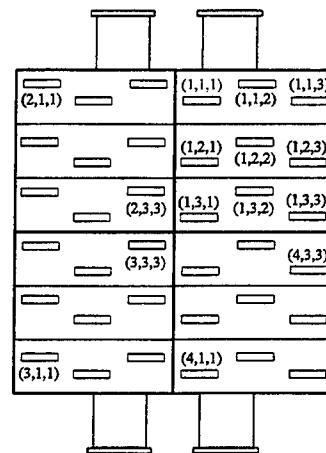


Fig. 5 Geometry of a  $4 \times 3 \times 3$  array.

Table I Slot offsets and slot lengths of radiating slots.

Slot index ( $s, t, n$ )	$x_{s,t,n}^{\text{off}}$ (mm)	$L_{s,t,n}$ (mm)
(1,1,1), (3,1,3)	$\mp 2.256$	16.758
(1,1,2), (3,1,2)	$\pm 3.015$	16.821
(1,1,3), (3,1,1)	$\mp 2.397$	16.779
(1,2,1), (3,2,3)	$\mp 2.859$	16.822
(1,2,2), (3,2,2)	$\pm 1.958$	16.287
(1,2,3), (3,2,1)	$\mp 2.221$	16.574
(1,3,1), (3,3,3)	$\mp 2.177$	16.842
(1,3,2), (3,3,2)	$\pm 1.516$	16.787
(1,3,3), (3,3,1)	$\mp 2.325$	16.830
(2,1,1), (4,1,3)	$\pm 2.848$	16.828
(2,1,2), (4,1,2)	$\mp 2.012$	16.630
(2,1,3), (4,1,1)	$\pm 3.138$	16.923
(2,2,1), (4,2,3)	$\pm 1.976$	16.437
(2,2,2), (4,2,2)	$\mp 2.679$	16.610
(2,2,3), (4,2,1)	$\pm 2.153$	16.500
(2,3,1), (4,3,3)	$\pm 1.969$	16.888
(2,3,2), (4,3,2)	$\mp 2.188$	16.756
(2,3,3), (4,3,1)	$\pm 1.603$	16.885

Table II Inclination angles and slot lengths of coupling slots.

Slot index ( $s, t$ )	$\theta_{s,t}$ (degrees)	$l_{s,t}$ (mm)
(1,1), (3,1)	$\pm 23.61$	16.986
(1,2), (3,2)	$\mp 21.47$	16.970
(1,3), (3,3)	$\pm 18.58$	16.948
(2,1), (4,1)	$\pm 24.73$	16.998
(2,2), (4,2)	$\mp 20.81$	16.964
(2,3), (4,3)	$\pm 17.88$	16.943

Due to the symmetry, the incident wave intensities were found to be  $a^{(s)}/a^{(1)} = 1$  for all sub-arrays.

The effect of the gradual inclusion of the mutual coupling is illustrated in Fig. 6, where the variation of the offset for the slot with index (1,3,2) as computed during the iterative steps is shown. The initial offset calculated for the case with zero mutual coupling is 3.082 mm, while the final offset is only 1.516 mm. The gradual introduction of the mutual coupling contributions over the next five iterative steps avoids abrupt changes in the slot dimensions, and convergence is easily achieved during the second half of the iterations.

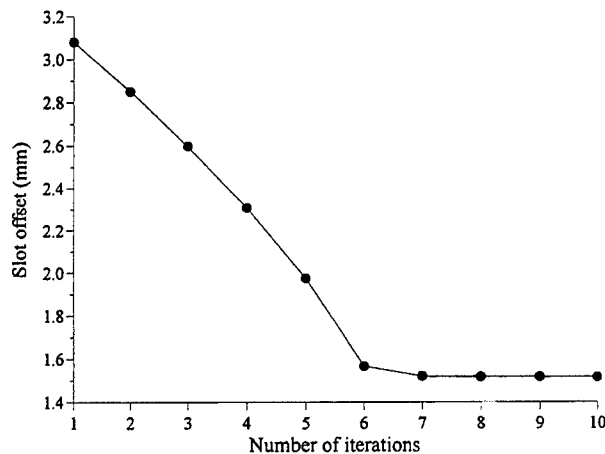


Fig. 6 Offset of slot (1,3,2) as computed during each iterative step.

## 8. CONCLUSION

The repeated computation of the external mutual coupling between slots, which involves numerical integration, is the most time-consuming part of the procedure. Utilization of an alternative expression for the coupling coefficient [10] results in an appreciable increase in efficiency. Some numerical experimentation may be required in order to obtain a design where all the slot offsets are within the desirable range. Slot offsets of less than  $w/2$  should be avoided. Care should also be taken not to allow the offsets to become too large, especially when using waveguides of reduced height. This may lead to problems related to the validity of the equivalent circuit models for the slots [11]. If the computed offsets of a particular branch are either too large or too small, the input conductance of those branches should be decreased or increased and the design repeated. Used in conjunction with an analysis procedure [12], the admittance levels of the branch lines may be chosen so as to optimize the off-centre frequency performance of the array.

The proposed algorithm is easily translated into computer code, and has been applied successfully to the design of arrays of varying sizes. The process of subdividing the nonlinear equations into smaller groups and gradually introducing the

effects of mutual coupling has proven to be very reliable when used in conjunction with reliable nonlinear equation solver routines.

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